The Bootstrap Variable Inductance
and its
Applications in AC Power Systems

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Abstract — **A new ac power controller is proposed, the Bootstrap Variable Inductance, which can emulate variable positive/negative inductance. The term 'reductance' is coined as a synonym for negative inductance. Unlike the conventional capacitive approach for treating unwanted inductive reactance, reductance has the advantage that resonance is impossible. The Bootstrap Variable Inductance has a variety of FACTS applications: series compensation of ac power lines, fault-current limiting, reactive-power control, and load power-factor improvement. We demonstrate its feasibility through simulations.**

I. INTRODUCTION

The principal aim of *flexible alternating-current transmission systems* (FACTS) [1] is to control the active and reactive power flow within an ac transmission system, while maintaining an adequate stability margin. Most of the system parameters are fixed by existing installations, system specifications and operating conditions. The usual approach is to add reactance in series with the lines, or in shunt at selected busbars. We present a new method for doing this, an electronically emulated variable inductance — the Bootstrap Variable Inductance. The effect of positive or negative inductance is produced by a bootstrap implementation involving a fixed inductance, a switched-mode power amplifier and storage capacitors. The technique can be applied in power systems for series line compensation, fault current limiting, shunt compensation, and load power-factor correction.

A. What is Negative Inductance?

First, let us examine the meaning of positive and negative inductance. In circuit-theory terms, inductance is defined by the branch equation $di/dt = v/L$. In other words, a positive voltage across the branch is associated with an increasing current through it. The constant of proportionality is the inductance, L henries. In the frequency domain, the branch impedance for sinusoidal excitation is $Z(j\omega) = V/I = j\omega L$. In an ac power system, inductive reactance at the synchronous frequency (power frequency) ω_s is $X_L = \omega_s L$, and is positive.

Now suppose that a positive voltage across a branch is instead associated with a *decreasing* current through it (ignoring for the moment how this could happen.) Then

$$\boxed{\frac{di}{dt} = \frac{-v}{\Gamma} \quad \text{defining equation for reductance, } \Gamma} \tag{1}$$

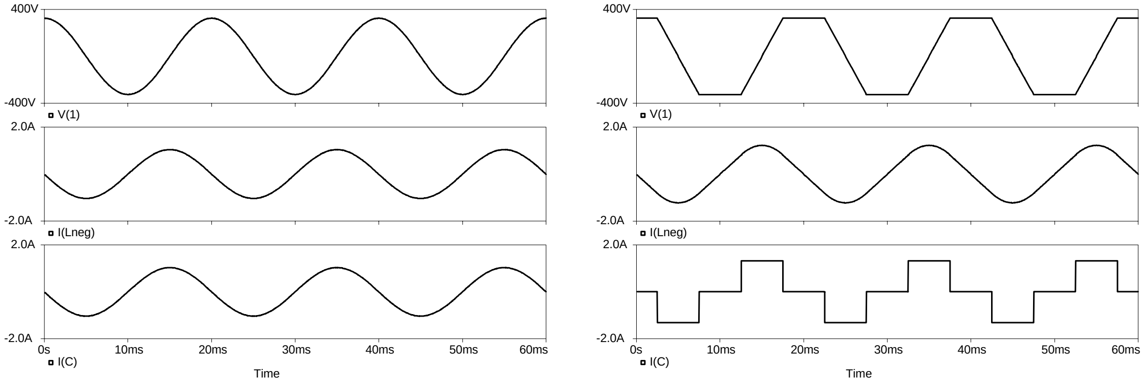
where Γ is a positive constant of proportionality, for which we propose the new term *reductance*.

Comparing with $di/dt = v/L$ we see that the branch has an inductance of $-\Gamma$, or equivalently, a negative inductance of Γ. To avoid confusion with negative quantities, we prefer to say it has a *reductance* of Γ henries, where Γ is positive: $\Gamma \equiv -L$. Reductance is a synonym for negative inductance. Its impedance is $Z_\Gamma(j\omega) = -j\omega\Gamma$. Its reactance at ω_s is $X_\Gamma = -\omega_s\Gamma$, which is negative, like that of capacitance.

B. Is Reductance Just Capacitance by Another Name?

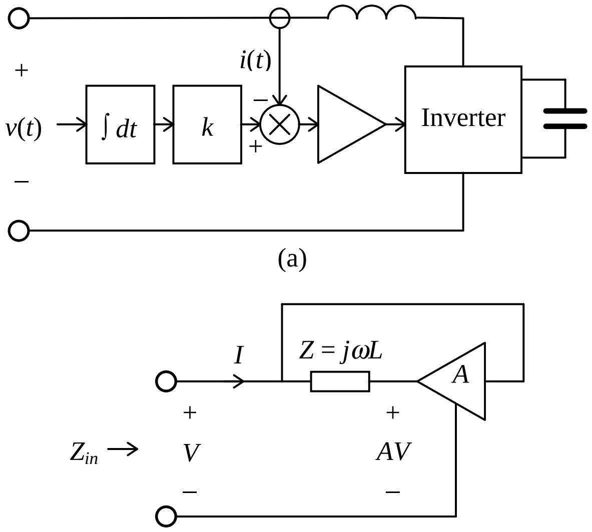
At the single frequency ω_s, reductive reactance is indistinguishable from capacitive reactance. As shown in Fig. 1(a), a sinusoidal voltage waveform results in identical sinusoidal current waveforms, leading by 90º. But when a range of frequencies is considered, reductive reactance $X_\Gamma = -\omega\Gamma$ differs greatly from capacitive reactance $X_C = -1/\omega C$. Both are negative, but as frequency increases, $|X_\Gamma|$ increases while $|X_C|$ decreases. For non-sinusoidal waveforms, which contain harmonic frequencies, their behavior is quite different. For example, the trapezoidal voltage waveform of Fig. 1(b) gives dissimilar currents. The current waveform for reductance is much smoother, with a lower harmonic content, suggesting that it would be more appropriate than capacitance if voltage distortion is present.

More importantly, capacitance will resonate with inductance, whereas reductance will not. This is because the total reactance of an LC loop is $\omega L - 1/\omega C$, which vanishes at the resonant frequency $\omega_0 = 1/\sqrt{LC}$. In the frequency domain this gives rise to rapid changes in impedance magnitude and



(a) (b)

Fig. 1: Comparison of reductance and capacitance. (a) Sinusoidal voltage waveform, (b) trapezoidal voltage waveform. Top: impressed voltage; middle: current in reductance; bottom: current in capacitance. Although reductance and capacitance are indistinguishable at a single frequency, when harmonics are present their behavior is quite different.



(a)

(b)

Fig. 2: Implementations of variable inductance/reductance: (a) VAPAR; (b) Bootstrap Variable Inductance (BVI).

phase close to ω_0; in the time domain ringing occurs at ω_0. In contrast, the total reactance of an $L\Gamma$ loop is $\omega(L - \Gamma)$, which is non-zero at all frequencies (assuming $\Gamma \neq L$). Thus we expect system stability to be improved when reductance is used in place of capacitance.

II. REALIZATION OF VARIABLE INDUCTANCE / REDUCTANCE

Negative inductance was first proposed as a power-system component in 1992, in the form of a *variable active-passive reactance* (VAPAR) [2], and this circuit has subsequently been developed by its inventors [3–7]. Fig. 2(a) shows the principle, based on feedback. A reference current is derived by integrating the terminal voltage. The terminal current is monitored and compared with this reference, and an inverter is controlled to drive the current error towards zero. The capacitor's dc voltage is set by a second feedback loop (not shown), which emulates the appropriate amount of parasitic resistance to maintain the voltage at its desired value.

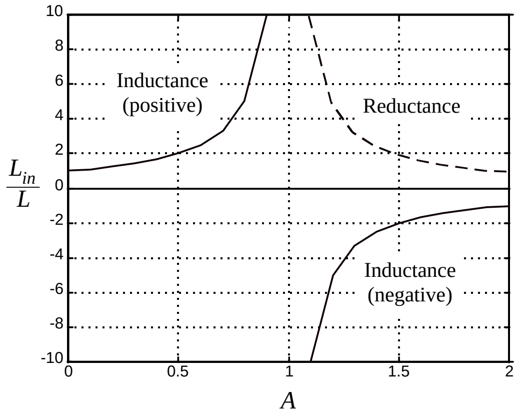


Fig. 3: Ratio of apparent input inductance L_{in} to physical inductance L as a function of amplifier gain A for the BVI. Positive reductance is identical to negative inductance, $\Gamma \equiv -L$.

In our approach we wish to reduce complexity, and in particular to avoid feedback with its concomitant stability problems. As an alternative to the VAPAR, we propose the *Bootstrap Variable Inductance* (BVI) of Fig. 2(b). The basic impedance conversion principle involved is that of bootstrapping, which may be described as a type of feedforward. The applied voltage V feeds an impedance $Z(j\omega)$ in series with a voltage amplifier of gain $A(j\omega)$. The effective input impedance is easily shown to be

$$Z_{in}(j\omega) = \frac{V}{I} = \frac{Z(j\omega)}{1 - A(j\omega)} \tag{2}$$

Now let $A(j\omega)$ be a positive constant A. When $A < 1$, Z_{in}

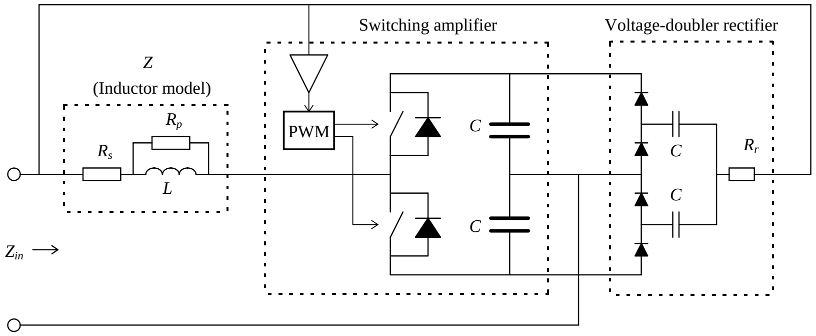


Fig. 4: Circuit schematic of the BVI, showing a possible implementation with self-powering from the terminal voltage.

has the same sign as Z but greater magnitude. When $A = 1$, $Z_{in} = \infty$ so $I = 0$. The input impedance Z_{in} appears effectively infinite because a unity-gain buffer drives its ‘neutral’ end to the same voltage as its ‘live’ end. This principle is known in analog electronics as *bootstrapping*. When $A > 1$, Z_{in} has the opposite sign to Z (negative impedance conversion); thus if we make Z inductive ($Z = j\omega L$), the circuit will emulate a reductive impedance ($Z_{in} = -j\omega\Gamma$). By varying A, a wide range of inductance and reductance can be obtained. For example, if A varies from 0 to 2, L_{in} ranges from L to ∞ and $-\infty$ to $-L$, as shown in Fig. 3, according to

$$\frac{L_{in}}{L} = \frac{1}{1-A} \tag{3}$$

Because feedback is not involved, only feedforward, the BVI is immune to instability — a valuable characteristic when the properties of the power system in which it is embedded can change. On the downside, the effective inductance L_{in} cannot be set precisely, as it is sensitive to variations in L and A. However, this should not be a problem in most applications.

III. PRACTICAL ISSUES

Our proposed circuit is shown in Fig. 4. As with any power electronic circuit, a number of issues must be resolved before the BVI can be considered a practical proposition.

A. *Amplifier Efficiency, Modulation and Switching Devices*

In principle, any type of power amplifier (e.g. Class AB) could be used for A. However, in high-power applications efficiency is paramount, so the amplifier must use a switched-mode output stage with pulse-width modulation (Class S) for 100% theoretical efficiency. Such amplifiers have been proposed for FACTS applications [8].

To allow versatility of modulation, the switching frequency of the BVI must be several times the power frequency. Various modulation strategies are possible, but comparison of these is outside the scope of the present paper. Here we assume naturally-sampled pulse width modulation (PWM) with a triangular carrier waveform at several times the power frequency; 1kHz is about the lowest practical switching frequency.

Ideally, the modulation scheme should be designed to minimize the harmonics introduced into the power system; to achieve this, the higher the switching frequency, the better. (Some work has been carried out by the authors in this direction, to be reported in a planned future paper.) On the other hand, the switching frequency must be low enough that switching losses are acceptable.

Conventional thyristor switches are not fast enough for these purposes, and the obvious choice at the megawatt power level is gate turn-off thyristors (GTOs). At lower power — tens of kilowatts, say — insulated-gate bipolar transistors (IGBTs) are another possibility.

B. *Voltage and Current Levels*

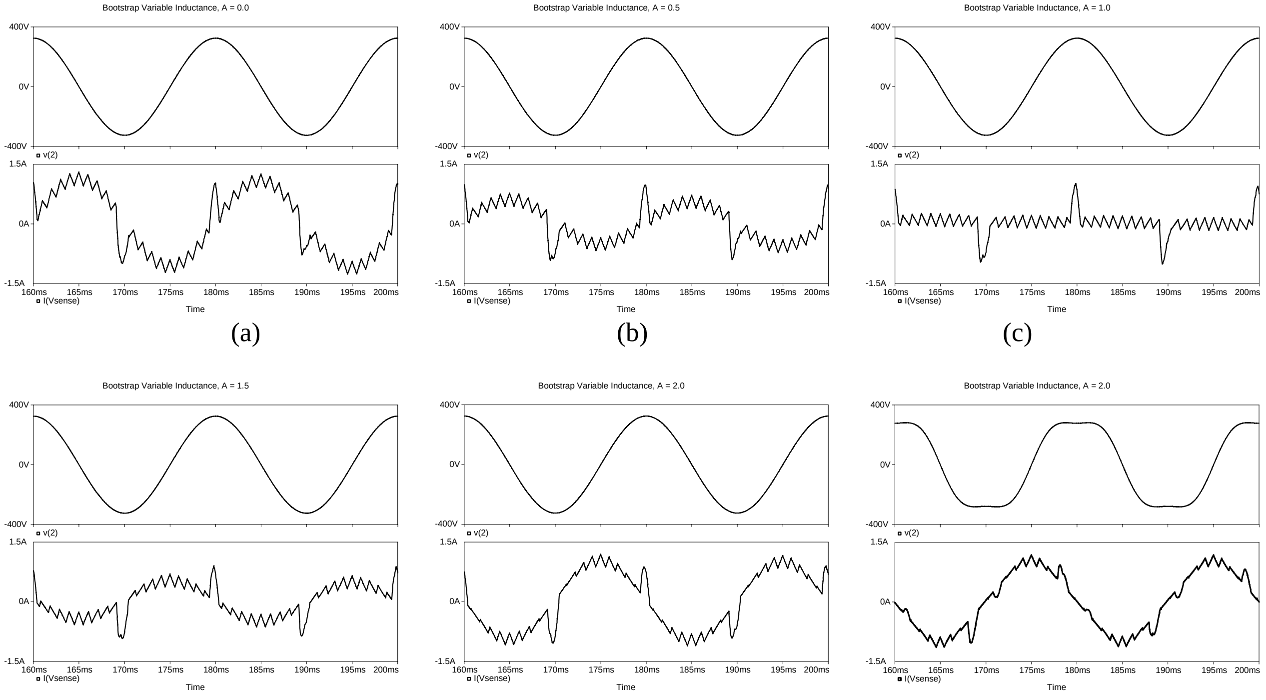
The voltage levels usually found in power systems (up to 1500kV) are incompatible with those of available GTOs and IGBTs (a few kilovolts). It would be possible to employ series-connected strings of devices, but in practice it is difficult to ensure equal voltage sharing both statically and dynamically.

To trade voltage for current, a step-down transformer can be used at the input of the BVI. This gives flexibility in other ways, to be discussed below. The resulting problem of current sharing is easier to solve than that of voltage sharing.

C. *DC Power Supply*

The power amplifier requires dc supply rails, and it is necessary to consider how much dc power is needed, and where it is to be drawn from. Now, inductance and reductance are non-dissipative circuit elements, which store energy. They alternately absorb and release energy, and the average power over a mains cycle is zero. In principle, therefore, the amplifier needs no dc power source, just a short-term energy reservoir, conveniently capacitance.

In practice, though, a small amount of dc power is needed to make up for unavoidable dissipation in the amplifier (conduction and switching losses) and in the inductor (core



(a) (b) (c)

(d) (e) (f)

Fig. 5: PSpice simulation results for the BVI of Fig. 4, including voltage-doubler rectifier. (The current spikes corresponding to peaks of the voltage waveform are the rectifier's charging current.) (a) A = 0.0, (b) A = 0.5, showing variable positive inductance; (c) A = 1.0, showing infinite inductance; (d) A = 1.5, (e) A = 2.0, showing variable reductance; (f) as (e) but with distorted voltage waveform.

and winding losses). One way to obtain the amplifier's dc supply rails is to feed a rectifier from the terminal voltage, effectively shunting Z_{in} with a nonlinear resistance. Because the only dc power required is that needed to supply the parasitic losses in the circuit, the additional current drawn should be relatively small if the efficiency is high.

To avoid saturation, the amplifier requires positive and negative rails at least A times the peak value of the terminal voltage V. If $A < 2$, a voltage-doubler rectifier can conveniently be used, as in Fig. 4. Alternatively, a simple rectifier fed from a step-up transformer or auto-transformer could be used. A third solution is to derive the dc from some auxiliary ac supply.

D. Inductor Losses

The inductor used for L will inevitably have parasitic losses, reducing the efficiency of the circuit. Moreover, when $A > 1$ these losses will appear as unwanted negative resistance in Z_{in}. This effect can be overcome by making A suitably frequency-dependent, as follows. Suppose the physical inductor can be adequately modeled by a pure inductance L plus series resistance R_s (representing winding loss) and parallel resistance R_p (for core loss). The amplifier gain required to nullify the resistances and produce an effective pure inductance of L_{in} can be shown to be

$$A(j\omega) = 1 - \frac{L}{L_{in}}\left(\frac{1}{1 + j\omega L/R_p} + \frac{1}{j\omega L/R_s}\right) \tag{4}$$

IV. SIMULATION RESULTS

To validate the concept of the BVI as a FACTS controller, PSpice simulations of the circuit of Fig. 4 were performed, with the switching amplifier powered by a voltage-doubler rectifier fed from the terminal voltage. The following parameters were used: V = 230V rms, 50Hz; L = 1H, R_s = 5Ω, R_p = 20kΩ; C = 100µF, R_r = 5Ω; PWM carrier = 1kHz symmetrical triangle wave. Figs. 5(a)–(e) show the applied voltage and the total current drawn (including that of the rectifier), for A = 0.0, 0.5 (varying inductance), 1.0 (infinite inductance) and 1.5, 2.0 (varying reductance) respectively. In each case the current lags or leads the voltage by close to 90º and has the magnitude predicted by analysis. (The superimposed current spikes are the rectifier's charging current.) Finally, Fig. 5(f) (with A = 2.0) shows a distorted voltage waveform containing 14% third harmonic, and the resulting BVI current waveform with just 1.9% third harmonic. Lowering of the harmonic percentage confirms that the BVI is emulating reductance rather than capacitance (which would accentuate harmonics). These simulations demonstrate that the BVI is a feasible stand-alone emulator of variable inductance and reductance.

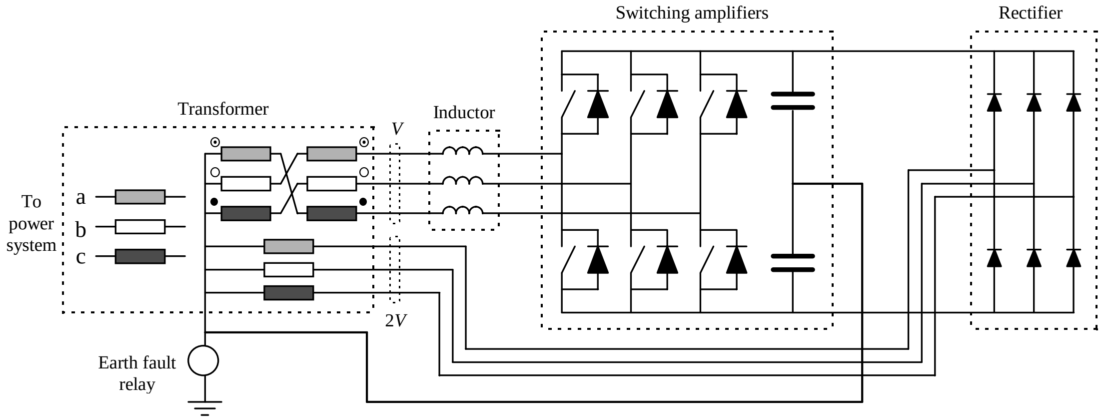


Fig. 6: Power circuit of a three-phase BVI. The step-down transformer has uncommitted primary windings and a zigzag-connected secondary. An additional low-power winding supplies the rectifier. Control circuits are not shown.

V. THREE-PHASE BVI

The BVI discussed above is a single-phase circuit, whereas in power systems applications three-phase operation is the norm. Although it would be possible to use three separate BVIs, it is natural to inquire whether there is any advantage to be gained by combining them into a single circuit.

A. Proposed Circuit

Fig. 6 shows our proposed three-phase BVI. Three single-phase circuits share a pair of dc capacitors. The three switching amplifiers are totally independent (i.e. three separate controllers are used, with no interconnection or switching synchronization); thus each phase is treated individually, only the dc circuit being shared. The midpoint of the two capacitors is connected as a return path to the ac source. This connection derives from the single-phase case, and is retained in the three-phase version so that unbalanced voltages can be tolerated, for greater robustness. Thus, for example, zero-sequence components at the input present no problems.

B. To Ground or Not?

The question arises whether the midpoint of the two capacitors should be grounded (i.e., connected to true earth) or left ungrounded as a floating neutral point.

It is paramount to protect the power system from any fault that might occur within the BVI. Suppose an earth fault were to occur somewhere in the BVI, bringing the voltage at the fault location close to zero. If the common point were not grounded, little or no fault current would flow. At first sight this might seem an advantage, but actually the reverse is true. An asymmetrical voltage condition would persist in the BVI, and this is undesirable for two reasons. First, some parts of the circuit would be at a higher potential above ground than with symmetrical operation. This would cause a higher electric field in their insulation and leakage paths, increasing the likelihood of breakdown. Second, the capacitances from the various switching devices to ground would be asymmetrical, giving rise to higher switching losses in one or more legs of the bridge, possibly leading to overheating and eventual failure. If the common point were floating, there would be no easy way to detect the fault and clear it from the system.

On the other hand, suppose the common point is grounded via a current-sensing relay as in Fig. 6. Then any earth fault can be detected, as the circulating fault current has to flow through the relay. Therefore we argue that a grounded common point is preferable, because it is possible to detect any earth fault and remove the faulty BVI from the power system before there is a chance of further disruption.

C. Transformer Winding Configuration

As indicated above, a step-down transformer is needed for voltage level adaptation. A transformer also brings the advantage of separating the BVI electrically from the power system. But how should the windings of the three-phase transformer be arranged?

First consider the primary winding. This is connected to the power system in two distinct ways (series or parallel), depending on the application of the BVI. For line-inductance compensation, the transformer primary is connected in series with the line, and must comprise three independent windings. In shunt applications, such as var compensation, the transformer primary is connected in parallel with the power system, similar to a load connection. In this case the primary windings could be configured either as a Y (wye, or star) or a Δ (delta). However, in practice a Y would be used at high voltages, since the windings' voltage stress is lower by a factor of $\sqrt{3}$.

Next, consider the secondary winding. Because the BVI circuit needs a common point, it is not possible to use a Δ configuration unless an artificial neutral point is constructed, causing additional problems of cost and dissipation. Nevertheless, it is undesirable to use a Y–Y configuration, because if the BVI generates triplen harmonics, they would be reflected directly to the power system. Moreover, these triplen

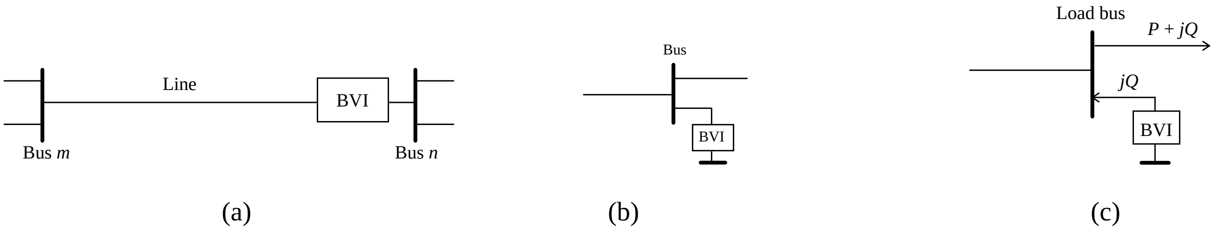


Fig. 7: One-line power-system diagrams showing power-systems applications of the BVI.
(a) Series compensation of line; (b) shunt compensation at bus (voltage regulation); (c) load power-factor correction.

harmonics would be in phase in all three arms of the transformer, causing the fluxes to be in phase (as zero-sequence components). This could cause excessive transformer iron loss and high external fields.

Suppose a Y–Y–Δ configuration were used, i.e. a tertiary Δ winding were added to cancel the effects of the triplen harmonics. This arrangement would satisfy both the common-point requirement and the triplen-harmonics requirement. But there are two disadvantages: first, the tertiary winding takes up winding space that could otherwise be occupied by the secondary, increasing its copper losses. Second, under fault conditions in the power system (e.g. an earth fault) the tertiary Δ winding is exposed to large zero-sequence currents.

A better solution is to use a zigzag secondary winding, as in Fig. 6. This not only provides a common point, but also cancels the effect of the triplen harmonics in each arm of the transformer core. The windings are all well utilized, and there are no problems with zero-sequence components caused by fault conditions in the power system. This is our preferred solution.

D. Combination of Transformer and Inductor

The BVI contains a low-frequency inductor of significant size, and is connected to the power system through a transformer. We believe it is possible to combine these two separate magnetic components into a single unit (integrated magnetics), lowering the cost. In principle, the inductor could be formed by leakage inductance referred to the secondary. For the transformer effect, a full-path core with the lowest possible reluctance is necessary; on the other hand, to achieve a given inductance, a suitable air gap is needed. Further discussion of this is outside the scope of this paper.

VI. APPLICATIONS IN AC POWER SYSTEMS

Unlike most FACTS controllers, the BVI finds a number of different uses in ac power systems. Here we suggest four such applications.

A. BVI as Series Compensator

Assuming negligible resistance, the active power transmitted through an ac transmission line is given by

$$P = \frac{|V_S||V_R|}{\omega_s L_{line}} \sin\delta \tag{5}$$

where V_S and V_R are the sending-end and receiving-end voltage phasors respectively, δ is the phase angle between V_S and V_R (the 'power angle'), ω_s is the synchronous frequency and L_{line} is the line inductance. In practice $|V_S|$, $|V_R|$ and ω_s are fixed by system specifications, L_{line} is a property which depends on the line geometry alone (at least for low frequencies, < 100kHz say), and δ is dictated by system configuration and stability requirements ($\delta \approx 28^\circ$ typically). The traditional way to increase the active power transmitted through a line is to place a capacitor in series with it, decreasing its effective reactance at ω_s. This is termed *series compensation*.

For more flexibility the effective capacitance is varied in steps, using a *thyristor switched series compensator* (TSSC), or continuously, using a *thyristor controlled series compensator* (TCSC). In any case, the added component injects a series voltage proportional to the line current but in quadrature with it ($V = I/j\omega C$). Unfortunately, the capacitance interacts with the system inductances to cause resonance, a potential source of system instability. This is because the total reactance is zero at $\omega_0 = 1/\sqrt{LC}$. Assuming $X_L + X_C > 0$ at ω_s (partial compensation), ω_0 is less than ω_s, typically around 15–30Hz. If ω_0 coincides with mechanical modes, e.g. shaft resonances, the resulting sub-synchronous resonance could be dangerous.

A second method of compensation has been proposed recently, the *static synchronous series compensator* (SSSC) [9]. Here a power converter synthesizes a series voltage independent of the line current's magnitude but in quadrature with it ($V \propto jI/|I|$). (Thus the effective series reactance introduced by the SSSC is inversely proportional to the line current.) Unlike capacitance, the SSSC does not cause resonance with system inductances. Moreover, while capacitive compensators can only increase the transmitted active power, the SSSC can either increase or decrease it.

However, it is our contention that the fundamental aim of compensation must be to change the effective *inductance* of the line. Contrast this with capacitive compensation, which affects the *reactance*, and the SSSC, which introduces a current-dependent *reactance*. Both are concerned with just a single frequency, ω_s, whereas we believe it is better to compensate over a substantial bandwidth.

To achieve this, series reductance Γ can be added by means of a BVI, as in Fig. 7(a). The total reactance is $\omega(L_{line} - \Gamma)$, which is non-zero at all frequencies (assuming $\Gamma \neq L$). Unlike capacitance, the BVI introduces no resonance. Thus we expect improved system stability. On the other hand, the BVI is similar to the SSSC in that it can either increase or decrease the transmitted power, by varying the effective value of L_{line}.

B. BVI as Fault Current Limiter

When a fault occurs in a transmission line, the current supplied to the fault location is mostly reactive, because the line and other power-system components are essentially inductive. Thus a fault causes a large reactive power flow. If a series-connected BVI is arranged to operate as a positive inductance under these conditions, it will limit the fault current.

However, this could interfere with the distance-protection relays normally employed with transmission lines. Distance protection measures the forward or backward impedance, which is proportional to the distance to the fault location. If an impedance is added in series with the transmission line, the distance relay could be overreached or underreached, causing an error in the relay measuring unit. For an important transmission line, the protection system and the BVI should be coordinated via a telecommunications link.

C. BVI as Shunt Compensator

There are three main reasons for controlling the reactive power flow in an ac power system. First, to improve synchronous-machine stability; second, to keep the voltage at the consumer close to its nominal value; and third, to flatten the grid-voltage profile, reducing unnecessary reactive power flow and lowering the transmission losses. Reactive power is usually managed by shunt compensation. Conventional methods include the *thyristor controlled reactor* (TCR), the *thyristor switched capacitor* (TCR) and the *static var compensator* (SVC), while a recent development is the *static synchronous compensator* (SSC). In all cases, the compensator draws a reactive (usually capacitive) current from the bus to regulate the bus voltage.

To act as a shunt compensator, the BVI can be connected across an ac power system bus, as in Fig. 7(b). Like conventional shunt compensators it draws reactive current, but because high-frequency PWM is used, the harmonic content is lower and more easily filtered. Moreover, unlike capacitive compensators, resonance is impossible, so there will be no amplification of existing harmonics.

D. BVI as Load Power-Factor Corrector

The loads in a power system are usually inductive, with a lagging power factor. (Induction machines represent a large proportion of the system's load.) A poor power factor means that considerable reactive power is drawn from the system, causing high currents and transmission losses. To improve the situation, large consumers are penalized for low power factor, and it is incumbent upon them to fit power-factor correction equipment, usually shunt capacitors. These can be placed at the supply connection point, or within individual equipment.

As before, it can be argued that inductance should be compensated by reductance, not capacitance. Again, reductance is advantageous in that it does not accentuate harmonic currents. Moreover, in the case of supply disconnection, capacitance could resonate with an inductive load to produce large overvoltages. There is no possibility of this occurring with reductance. Fig. 7(c) shows the connection.

VII. CONCLUSION

A new way has been proposed for emulating variable inductance/reductance in high-power applications, the Bootstrap Variable Inductance. Its validity has been demonstrated via simulation, including self-powering from its terminal voltage. The BVI has many practical applications in ac power systems, where it may be used to compensate for the inductance inherent in power-system components. This is preferable to using capacitance because resonance is impossible, and improved power-system stability should result. It is hoped that the BVI will take its place alongside other FACTS controllers, and may find further applications at lower power levels in ac power systems.

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