POWER ELECTRONICS:
A FIELD RICH IN NONLINEAR DYNAMICS

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This review paper starts by setting out the aims and applications of power electronics, and continues with a brief history and a list of the important power semiconductor devices. The related areas of ac machines and power systems are also briefly visited. The development of nonlinear dynamics in electronic circuits is reviewed. Then a typical power converter, a controlled buck dc-dc converter, is modelled by the conventional method of averaging and linearisation (which predicts stability), and by a nonlinear map based method, which reveals bifurcations, subharmonics and chaos. The numerical problems caused by the discontinuities in the state equations of power electronics are discussed. Finally, some possible future applications are considered.

1 Introduction to Power Electronics

Most branches of electronics are concerned with processing information or signals; in contrast, *power electronics* deals with the processing of electrical energy. Power converters do not have an end of their own, but are always an intermediary between an energy producer and an energy consumer. The field is one of growing importance: it is estimated that by 2000, over half the electrical energy generated will be processed by power electronics before its final consumption, a proportion that is likely to reach 90% during the next century.

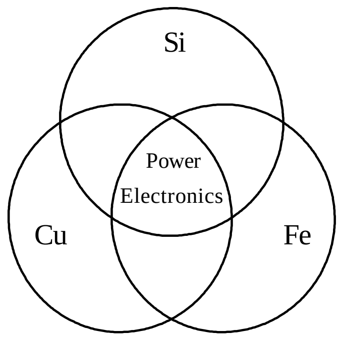

Fig. 1: The elements of power electronics.

As illustrated by Fig. 1, power electronics involves the interaction of three elements: copper, which conducts electric current; iron, which conducts magnetic flux; and, in prime position, silicon. This is used not only in the semiconductor devices that handle the power conversion, but also in the increasingly sophisticated circuitry that controls them. It is the inclusion of active semiconductor devices that distinguishes power electronics from electrical engineering, which is concerned essentially with applications of copper and iron. Unlike other areas of high power analogue electronics, power electronics uses the semiconductor devices as switches.

Power electronics is a “green” technology, with three main aims:

- To convert electrical energy from one form to another, facilitating its regulation and control.
- To achieve high conversion efficiency and therefore low waste heat.
- To minimise the mass of power converters and the equipment (such as motors) that they drive.

Intelligent use of power electronics will allow consumption of electricity to be reduced. Hence the rate of fossil fuel depletion may be slowed, and the associated problem of global warming eased. Minimisation of mass means a reduction in the material and energy resources required for manufacture and use. Mass reduction is especially important for aerospace and vehicular applications, where it translates into lower fuel consumption.

Since electrical power supplies can be either dc or ac, there are four basic types of power converter: ac-dc converters (rectifiers), dc-ac converters (inverters), dc-dc converters, and ac-ac converters. Power electronics technology is increasingly to be found in the home and workplace [1]–[3]: familiar examples are the domestic light dimmer, and the switched mode power supplies of personal computers. Fields in which power electronics has been applied include:

- Heating and lighting control
- Drives for industrial motion control
- Battery chargers
- Solid state relays and circuit breakers
- Fluorescent lamp ballasts
- Induction heating
- Traction applications such as locomotives
- Off-line dc power supplies

- Spacecraft power systems
- Switched mode audio amplifiers
- Electric power transmission
- Uninterruptible power supplies (UPS)
- Conditioning for alternative energy sources
- Automobile electronics and electric vehicles

2 History of Power Electronics

Prior to 1900, the only method of converting electrical energy from one form to another was by means of rotating machines. Mercury arc rectifiers were introduced around 1900, making *static power conversion* possible. Solid state copper oxide rectifiers appeared in 1926 and selenium rectifiers a year later. By the 1930s a third electrode had been added to mercury arc rectifiers, allowing control of the rectification process by delaying the onset of conduction. In the 1940s, an early form of switched mode power supply appeared: electromechanical vibrator choppers with synchronous rectification, providing the anode supply for car radios and other portable equipment. Post-war development led to the invention of the transistor in 1948. Group IV semiconductor materials, germanium and silicon, were applied to produce pn junction power diodes, which became commercially available in the 1950s. After a period of neglect, controlled mercury arc rectifiers enjoyed a renaissance in the 1950s when they were applied to high voltage dc (HVDC) links between ac power systems.

In 1956 the first experimental thyristor was made, and in 1958 General Electric announced a commercial silicon controlled rectifier (SCR), an important member of the thyristor family. The SCR was a practical replacement for the controlled mercury arc rectifier, and its invention made solid state power conversion possible.

The term *power electronics*, at first synonymous with thyristor electronics, did not come into general use until the 1970s. There was no journal expressly devoted to the field until the *IEEE Transactions on Power Electronics* commenced publication in 1986. Although it has nearly a century of history behind it, power electronics has only recently come of age.

3 Power Switching Devices

The key to power electronics is the availability of suitable switching devices. The ideal switch would pass no current when off, drop zero voltage when on, transfer instantaneously between its on and off states, and have unlimited voltage and current capability. The shortcomings of real devices impose limits on the development of power converters. The main types are listed below.

Diodes: Diodes may be thought of as passive switches, or “non-return valves”. The types currently available include fast recovery pn junction, p-i-n, and Schottky diodes. The latter have low conduction loss and negligible charge storage, and are widely used at low voltages.

Thyristors: The SCR is a pnpn device. When reverse biased, it blocks the flow of current; when forward biased, conduction is inhibited until a trigger pulse is applied to the gate. The SCR then conducts until the current through it falls to zero, whereupon it resumes blocking. Modern variants include asymmetric SCRs, reverse conducting thyristors, and gate turn-off thyristors. Being rugged devices available in high ratings, thyristors have been widely applied up to extremely high power levels, e.g. in the 2GW England–France HVDC link. Most types are rather slow, limiting their applications to low frequencies.

Bipolar junction transistors: Silicon bipolar junction transistors (BJTs) were developed during the 1960s, and by the 1970s were employed in switched mode power supplies. BJTs are minority carrier devices, so speed is a limitation: practical switching frequencies are limited to around 40kHz.

Power MOSFETs: The power MOSFET (metal oxide semiconductor field effect transistor) became a commercial proposition in the early 1980s. A majority carrier device, it is capable of switching at megahertz frequencies, but contains a slow parasitic body–drain diode. MOSFETs are replacing BJTs in low power applications such as switched mode power supplies. The MOSFET’s construction is not suitable for very high powers, and voltage ratings are lower than for competing devices.

IGBTs: The insulated gate bipolar transistor (IGBT) became a commercial reality in the late 1980s. It acts like a MOSFET driving a power BJT, and has some of the advantages of both: ease of drive, high ratings and low conduction loss. But minority carrier charge storage makes the IGBT turn off with a long “current tail”, causing high switching loss. IGBTs are widely used in motor drive applications.

Apart from voltage and current ratings, the main limitation of most power devices is their switching speed. There is constant pressure towards higher operating frequencies, in pursuit of smaller inductors, capacitors and transformers. Unfortunately, switching losses and other losses increase, imposing an effective upper limit for a given device and circuit.

4 Development of Chaos in Electronic Circuits

Nonlinear dynamics has developed in parallel with power electronics, but the two fields have begun to converge.

The history of chaotic dynamics can be traced back to the work of Poincaré on celestial mechanics around 1900. However, the first inkling that chaos might be important in a real physical system was given by Lorenz in 1963 [4], who discovered the extreme sensitivity to perturbations of a simplified computer model of atmospheric convection. Lorenz's paper, which appeared in an obscure journal, was largely overlooked for some years. In 1976, May published an influential article [5] describing how simple nonlinear systems can have complex, chaotic behaviour. In the late 1970s, Feigenbaum [6] analysed the period-doubling cascades that form one of the commonest routes to chaos.

Chaotic effects in electronic circuits were first noted by Van der Pol in 1927 [7], [8]. A relaxation oscillator comprising a battery, a neon bulb, a capacitor and a resistor, was driven by a 1 kHz sinusoidal signal and tuned to obtain subharmonics, but "an irregular noise" was often heard. There was little interest in explaining such spurious oscillations for about 50 years. In 1980 Ballieul, Brockett and Washburn [9] suggested that chaos might occur in dc-dc converters and other control systems incorporating a pulse-width modulator. In 1981 Linsay published the first modern experimental report of electronic chaos [10]: a driven resonant circuit, employing a varactor diode as a nonlinear capacitor. In 1983 Chua synthesised the first autonomous chaotic electronic circuit [11], the double scroll oscillator, now usually known simply as *Chua's circuit*, which has been widely studied as the archetypal chaotic electronic circuit [12].

In 1984, Brockett and Wood [13] presented a conference paper mentioning chaos in controlled buck dc-dc converter. A 1988 letter by Hamill and Jefferies [14] appears to be the first *analysis* of chaos in power electronics. Wood further described chaos in a switching converter at a 1989 conference [15], and in the same year a paper by Deane and Hamill [16], [17] identified several other ways by which chaos might arise in power electronics. These ideas were further developed in [18]–[22], which are mainly concerned with prediction and experimental confirmation of chaos in dc-dc converters under various control schemes. In the mid 1990s this work is starting to be built upon by other researchers [23]–[33]. It is still considered exotic by the mainstream power electronics community.

5 Power Converters

Since the object is to convert electrical energy at high efficiency, the ideal power converter would contain only lossless components. Two basic groups that can be approximated by real components are available:

- Switching components, such as transistors and diodes. An ideal switch is either on ($v = 0$) or off ($i = 0$), so its vi product is always zero and it never dissipates energy. Active switches such as transistors turn on and off in response to an applied signal; passive switches (diodes) have a highly nonlinear v–i characteristic.
- Reactive (energy storing) components, such as inductors and capacitors. They are characterised by differential equations, $v = L\, di/dt$ for an inductor, $i = C\, dv/dt$ for a capacitor. It can be shown that they absorb energy from a circuit, store it and return it.

Power converters employ components from both groups. Energy is pumped around the circuit by the switching components, while the reactive components act as intermediate energy stores and input/output reservoirs. The presence of both types of component implies that the circuits are *nonlinear, time varying dynamical systems*. Anyone familiar with nonlinear dynamics will appreciate that power converters are difficult to analyse, and are likely to show a wealth of curious behaviour.

There are also several unavoidable sources of unwanted nonlinearity in practical power converters:

- The semiconductor switching devices have intrinsically nonlinear dc characteristics.
- They also have nonlinear capacitances, and most suffer from minority carrier charge storage.
- Nonlinear inductances abound: transformers, chokes, magnetic amplifiers, and saturable inductors used in snubbers.
- The control circuits usually involve nonlinear components: comparators, PWMs, multipliers, phase-locked loops, monostables and digital controllers.

The driven *R-L-D* circuit [10] has a close relative in power converters: when a transformer feeds a rectifier diode, the leakage inductance resonates with the diode's nonlinear capacitance to give a chaotic transient when excited by the switches. A similar effect is ferroresonance: a tuned circuit involving a saturating inductor

[34]–[38]. This too has practical relevance: it is exploited to regulate voltages, but unintended ferroresonance in power systems can cause excessive voltages and currents.

6 Related Areas

Adjustable speed drives are a rapidly growing market for power electronics. Here, power converters are combined with electric motors and sophisticated control electronics. The main thrust of current work is to replace conventional dc drives with ac drives. Dc motors are easy to control for a good dynamic response, but have a complex physical construction and a poor power-to-weight ratio. They utilise a commutator and brushes, which cause sparking and radio interference, and are subject to mechanical wear. Much research has been done into supplying and controlling ac machines such as squirrel-cage induction motors, permanent-magnet synchronous motors, "brushless dc" motors and switched reluctance motors. These machines are mechanically simple and are therefore inexpensive and reliable, but they are difficult to control if variable speed and rapid dynamic response are required. The power electronics and digital control techniques being applied are an excellent example of the ascendancy of silicon over copper and iron.

Unfortunately, ac motors are themselves inherently nonlinear. For example, the induction motor may be modelled by a nonlinear and highly interactive multivariable structure. It is the task of vector control techniques to unravel this model, decouple the flux and torque variables, and allow a relatively simple outer control loop. Another example is the switched reluctance motor, in which the self and mutual inductances vary not only with the shaft rotation, but also with saturation of the magnetic path — which itself depends on the shaft position as well as the drive waveform. Finally, the permanent magnet stepper motor, operated open loop with an inertia load, exhibits bifurcation from steady rotation to chaotic back-and-forth juddering, a phenomenon that has been well known for many years but little studied. Combining switched circuits and nonlinear electromechanical devices, adjustable speed drives would seem to be a fruitful source of nonlinear behaviour and, because of their importance to industry, an appropriate subject for detailed study.

The field of *electric power systems* deals with the generation, transmission and distribution of 50/60Hz power. Bifurcation theory has been applied successfully to simple models of power systems [39]–[45], and can help explain undesirable low frequency oscillations (sub-synchronous resonances) and voltage collapses.

Power systems are finding increased use of power electronics. In developed countries, about 60% of electricity generated is used to power motors, and a further 20% is consumed by lighting; as power electronics penetrates these areas, more and more power converters will be connected to the ac supply. Furthermore, power electronics is increasingly being used by the utility operators themselves to process power on a large scale. Widespread use of megawatt power converters in flexible ac transmission systems (FACTS) is anticipated. In order to maximise the capacity and cost effectiveness of existing power systems as demand rises, progressive interlinking is taking place on a continental level. Undesirable nonlinear effects are likely, unless precautions are taken to study them. It is to be hoped that catastrophic bifurcations, such as the one leading to the north-east US power failure of 1965, can be avoided!

7 The Buck Dc-Dc Converter

As a concrete example of a power converter, an example is now presented, for which the conventional modelling and control approach is contrasted with one derived from nonlinear dynamics. The subject is one of the simplest but most useful power converters, the buck dc-dc converter, a chopper circuit that converts a dc input to a dc output at a lower voltage. (Many switched mode power supplies employ circuits closely related to it.) An application of current importance is conversion of the standard 5V dc supply used in computers to the 3.3V needed by a Pentium CPU chip. A buck converter for this purpose can achieve a practical efficiency of 92%, whereas a linear regulator would be only 66% efficient — producing six times as much waste heat. Although this example is at a low power level, buck converters are also used at several kilowatts.

The basic open loop buck converter is shown in Fig. 2. The switch S opens and closes periodically at the switching frequency f_s, with a duty factor d. When S is closed, the input voltage V_I is transferred to the LC low pass filter. When S is open, the inductor maintains its current i, forcing the diode D to conduct and ground the input of the LC filter. Thus the filter sees a square wave between 0 and V_I. The cut-off frequency of the filter is much lower than f_s, removing most of the switching ripple and delivering a relatively smooth output voltage v to the load resistance R. The output can be varied by changing the duty factor d, i.e. by pulse width modulation (PWM).

The operation described is known as *continuous conduction mode* (CCM), since the inductor passes current without a break. However, if the output is only lightly loaded, the inductor current can become zero for part of

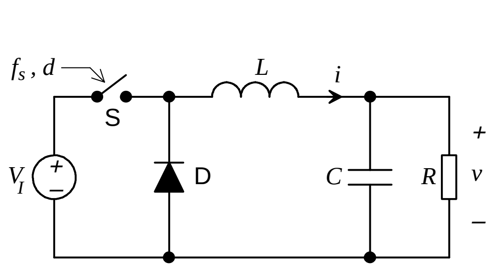

Fig. 2: Open loop buck dc-dc converter.

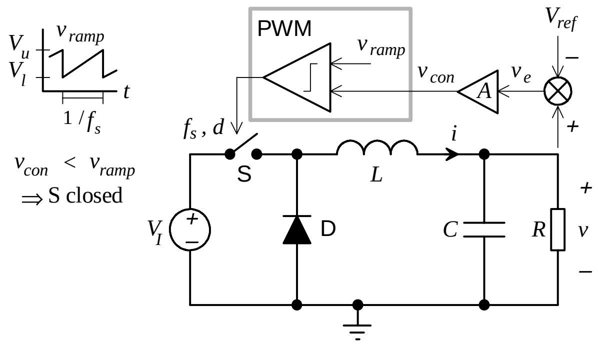

Fig. 3: Buck converter with proportional closed loop controller.

the cycle as D comes out of conduction: *discontinuous conduction mode* (DCM). (The terms “continuous” and “discontinuous” are used in a non-mathematical sense here.)

In practice it is necessary to regulate v against changes in the input voltage and the load current, by adding a feedback control loop as in Fig. 3. In this simple proportional controller, a constant reference voltage V_{ref} is subtracted from the output voltage and the error, v_e, is amplified with gain A to form a control signal, $v_{con} = A(v - V_{ref})$. This signal feeds a simple PWM circuit comprising a ramp (sawtooth) oscillator of frequency f_s and voltage $v_{ramp} \in [V_l , V_u]$, and a comparator which drives the switch. This conducts whenever $v_{con} < v_{ramp}$; thus v_{con} determines d. The intended mode of operation is a steady state in which the output voltage stays close to V_{ref}.

8 Conventional Model of the Buck Converter

The conventional way of modelling this type of circuit is to take an average over a switching cycle, an approach first proposed by Wester [46]. Since conventional control theory requires a linear model, the averaged circuit is generally linearised about a suitable operating point. *State space averaging*, developed by Ćuk [47], [48], operates on the state equations of the circuit. An alternative method uses “injected and absorbed” currents [49]. Vorpérian [50], [51] gave a method of treating the switch–diode combination in isolation from the converter circuit. Regardless of the details, these methods have the same aim: to replace the nonlinear, time varying dynamical system with an averaged, linearised one. (The justification is that when designing the control circuit, one need no longer be concerned with the microscopic details of the power switching.) Clearly, something must be lost in the process.

8.1 Continuous Conduction Mode

The state space averaging approach will be demonstrated for CCM. The state equations are:

$$\frac{di}{dt} = \begin{cases} (V_I - v)/L & \text{S conducting, D blocking} \quad (1a) \\ -v/L & \text{S blocking, D conducting} \quad (1b) \\ 0 & \text{S and D both blocking} \quad (1c) \end{cases} \tag{1}$$

and

$$\frac{dv}{dt} = \frac{i - v/R}{C} \tag{2}$$

Averaging: In CCM, S conducts for a fraction d of each cycle and D conducts for the remainder, $1 - d$; (1c) is not involved. The averaged equations are found by multiplying (1a) by d and (1b) by $1 - d$, and summing:

$$\frac{di}{dt} = (d\, V_I - v)/L \tag{3}$$

$$\frac{dv}{dt} = \frac{i - v/R}{C} \tag{4}$$

In this simple example, only one of the state equations is affected: (2) comes through the averaging process unaltered. This may not be the case with other converters.

Perturbation: Let each quantity comprise a constant (dc) component, represented by an upper case symbol, and a small perturbation component, represented by a lower case symbol with a circumflex. Thus, for instance, let $i(t) =$

$I + \hat{i}(t)$. Doing this for i, v and d, substituting into (3) and (4), and using the fact that $dI/dt = 0$, $dV/dt = 0$ (I and V are constants), the following expressions are obtained:

$$\frac{d\hat{i}}{dt} = \frac{\left(D + \hat{d}\right)V_I - (V + \hat{v})}{L} \tag{5}$$

$$\frac{d\hat{v}}{dt} = \frac{I + \hat{i} - (V + \hat{v})/R}{C} \tag{6}$$

Steady state: To find the steady state (the equilibrium point of the averaged dynamical system), all perturbation terms are set to zero, and the LHS of each state equation is also set to zero. This results in

$$V = DV_I \quad \text{and} \quad I = V/R \tag{7}$$

which accord with an intuitive understanding of the circuit's operation.

Linearisation: Finally, the system is linearised about this steady state operating point. Expanding (5) and (6), neglecting second order perturbation terms (in this particular case there are none, but had we had written $v_I(t) = V_I + \hat{v}_I(t)$ the term $\hat{d}\,\hat{v}_I$ would have arisen), and subtracting away the respective steady state equations of (7), the following are obtained:

$$\frac{d\hat{i}}{dt} = \frac{V_I\hat{d} - \hat{v}}{L} \tag{8}$$

$$\frac{d\hat{v}}{dt} = \frac{\hat{i} - \hat{v}/R}{C} \tag{9}$$

These linear differential equations represent the small signal (ac) behaviour of the buck converter.

Transfer functions: Laplace transforms of (8) and (9) are taken by writing s for d/dt. Eliminating $\hat{i}$ between the two transformed equations yields the control-to-output transfer function:

$$\frac{\hat{v}}{\hat{d}} = \frac{V_I}{1 + sL/R + s^2LC} \tag{10}$$

where $\hat{v} = \hat{v}(s)$ now means the Laplace transform of $\hat{v}(t)$, etc. This transfer function forms part of the feedback loop and determines the closed loop stability. Using a similar averaging approach, the transfer function of the error amplifier and PWM is easily found as

$$\frac{\hat{d}}{\hat{v}_e} = \frac{A}{V_u - V_l} \tag{11}$$

Hence the overall loop gain is

$$G(s) = \frac{\hat{v}}{\hat{v}_e} = \frac{AV_I}{V_u - V_l} \cdot \frac{1}{1 + sL/R + s^2LC} \tag{12}$$

Stability: Equation (12) describes a standard second order system, with dc gain $AV_I/(V_u - V_l)$, undamped natural frequency $\omega_n = 1/\sqrt{LC}$ and damping factor $\zeta = \sqrt{L/4CR^2}$. Given R, the values of L and C are chosen by the designer on power considerations: L is made large enough to ensure CCM operation, and C is chosen to give an acceptably small output voltage ripple. This generally results in an underdamped response ($\zeta < 1$) with $\omega_n/2\pi << f_s$.

Consider the example of a buck converter designed to accept an input voltage of 15V to 40V and produce a regulated output voltage close to 12V [18], [20], [32]. The following parameter values apply: f_s = 2.5kHz, A = 8.4, V_u = 8.2V, V_l = 3.8V, V_{ref} = 12V, L = 20mH, C = 47µF and R = 22Ω; therefore $\omega_n/2\pi$ = 164Hz and ζ = 0.47. The system's phase margin can be calculated from (12) by setting $s = j\omega$, determining the unity gain frequency ω_1 at which $|G(j\omega_1)| = 1$, finding the phase angle $\angle G(j\omega_1)$, and adding 180°. The phase margin varies from 10.2° at the minimum input voltage of 15V to 6.2° at the maximum, 40V. These margins are rather small: greater than 30° would be desirable. Nevertheless, according to the average model, the closed loop converter is stable over the entire input voltage range.

8.2 Discontinuous Conduction Mode

Analysis of operation in DCM is more complicated, because there are now three circuit configurations during a cycle: S conducts for a fraction d of each switching cycle; D conducts for a time that depends on the circuit action and ceases when i = 0; and both S and D block for the remaining time. Thus equations (1a)–(1c) are all involved, together with a condition determining D's conduction interval (found by assuming that i follows a

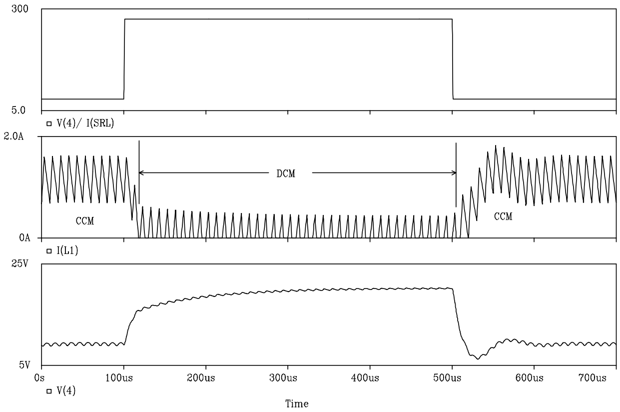

Fig. 4: Transient response of open loop buck converter, showing first order characteristics in DCM and second order in CCM. Top to bottom: Load resistance *R*; inductor current *i*; output voltage *v*.

straight line). Despite the increased complexity, similar principles can be applied as for CCM. The control-to-output transfer function is found to be of the form

$$\frac{\hat{v}}{\hat{d}} = \frac{A_0}{1 + s\tau} \tag{13}$$

where the dc gain A_0 is a function of f_s, L, R and D, and the time constant τ is

$$\tau = \frac{CR}{2}\left(1 - \frac{1}{\sqrt{1 + 8f_sL/RD^2}}\right) \tag{14}$$

Note that the DCM model is of first order, not second order as might be expected. An explanation is that the inductor does not really enter into the long-run dynamics of the system. By definition i is zero at both the start and the finish of every cycle; the role of L is simply to set the amount of charge transferred from V_I to C. The change of order can be seen in the simulation of Fig. 4, in which the load resistance R is stepped so that the CCM/DCM boundary is crossed. The pole at $s = -1/\tau$ is not fixed, but varies with the operating point.

Since a first order system with proportional control has a phase margin greater than 90°, its stability is expected to be extremely good. (However, Vorpérian [51] has argued that there is actually a second pole at a high frequency, which reduces the phase margin.)

9 Nonlinear Map Based Model of the Buck Converter

No method that relies upon linearisation is able to predict effects such as subharmonics and chaos, which are peculiar to nonlinear systems. Nonlinear dynamics offers an alternative approach [20]. Here the full detail of the switching operations is retained, so the model is likely to be more accurate; but inevitably, the description will be more complex.

9.1 Continuous Conduction Mode

The aim is to find a two-dimensional mapping **F**: $\text{Å}^2 \rightarrow \text{Å}^2$ which describes how the state vector $\mathbf{x} = [i \;\; v]^T$ evolves from one ramp cycle to the next: $\mathbf{x}_{m+1} = \mathbf{F}(\mathbf{x}_m)$. Steady state period-1 operation corresponds to a fixed point of the map, $\mathbf{x}^* = \mathbf{F}(\mathbf{x}^*)$. As before, it will be assumed that the converter always operates in CCM, and that the filter network is underdamped.

With S closed, equations (1a) and (2) govern the time evolution of the state vector. Their solutions may be written

$$i = \exp\frac{-t}{2CR}\cdot(a_1\sin\omega_d t + b_1\cos\omega_d t) + \frac{V_I}{R} \tag{15}$$

$$v = \exp\frac{-t}{2CR}\cdot(a_2\sin\omega_d t + b_2\cos\omega_d t) + V_I \tag{16}$$

where $\omega_d = \sqrt{\frac{1}{LC} - \frac{L}{C^2R}}$ and a_1, b_1, a_2, b_2 are constants derived from the initial conditions. The state vector follows (15) and (16) until the switching condition $v_{con} = v_{ramp}$ is satisfied. Then S opens, at the switching instant $t = t_s$. For this circuit the state vector is continuous (in the mathematical sense), so the final values of i and v for one interval become the initial values for the next. With S open, (1b) and (2) govern the motion. Resetting t to zero, the solutions are now

$$i = \exp\frac{-t}{2CR}\cdot\left(a_1'\sin\omega_d t + b_1'\cos\omega_d t\right) \tag{17}$$

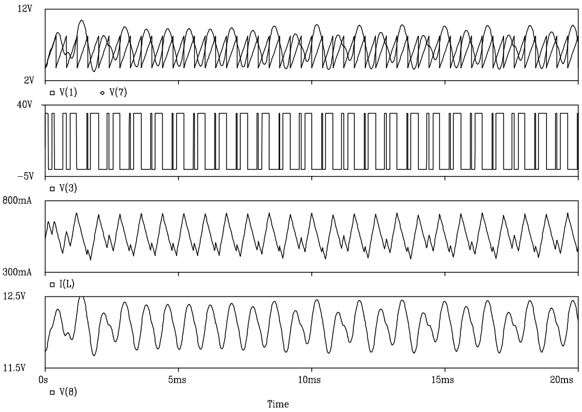

Fig. 5: Simulated chaotic waveforms for the buck converter in CCM
with V_I = 35V. Top to bottom: v_{ramp} and v_{con}; voltage across D;

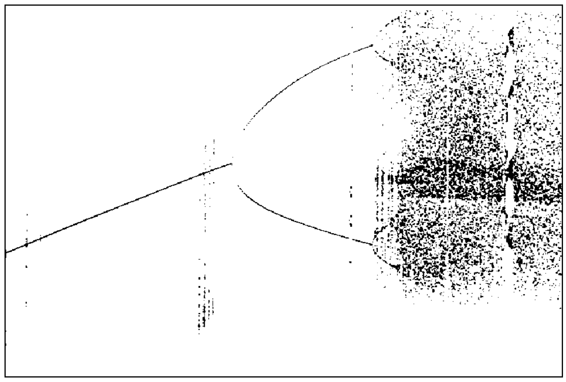

Fig. 6: Simulated bifurcation diagram for the buck converter in CCM:
{ i_m } plotted against $V_I \in$ [15V, 40V].

$$v = \exp \frac{-t}{2CR} \cdot \left(a_2' \sin \omega_d t + b_2' \cos \omega_d t \right) \tag{18}$$

where the new constants a_1', b_1', a_2', b_2' can be calculated from a_1, b_1, a_2, b_2 .

This process of alternating switch transitions, applied over the ramp cycle $t \in [0, 1/f_s)$, defines the mapping **F** that takes $\mathbf{x}_m$ to $\mathbf{x}_{m+1}$. Unfortunately, there is a snag: finding the set of switching instants { t_s }. Switching occurs whenever $A(v - V_{ref}) = v_{ramp}$, and this introduces two problems. First, because $v(t)$ is a damped sinusoid, finding the switching instants involves solving a transcendental equation, which must be done numerically. Second, there is no guarantee that the switch will close and reopen exactly once in every ramp cycle. In fact, it turns out that the switch can operate *any* number of times, from zero to infinity. (In practice there is an upper bound, set by parasitic effects.) There is no known way to predict the number of switching operations for a particular ramp cycle. Different types of operation follow each other in a sequence that may repeat or not, depending on parameter values. Subharmonics and chaos are thereby possible. Fig. 5 shows typical simulated waveforms; similar ones were observed in experiments.

Although this converter is not susceptible to much further analysis, the mapping **F** can be expressed as a deterministic algorithm that allows numerical investigations. (For certain other converters, it is possible to obtain an analytical mapping (at least approximately) and hence prove the operation to be chaotic [19].)

Because it is easy to vary, the input voltage V_I was chosen as the bifurcation parameter. The inductor current i was sampled at the start of every ramp cycle and plotted as the bifurcation diagram of Fig. 6, which was obtained from simulations. A period doubling route to chaos is visible. Fig. 7 shows a Poincaré section of the strange attractor associated with this circuit. Experimental measurements were in excellent agreement with the simulations [20].

9.2 Discontinuous Conduction Mode

DCM operation of the buck converter has been studied by Tse [25]. If the state vector is sampled at the start of each ramp cycle, the discrete system is truly one-dimensional. Since $i \equiv 0$ at every sample (assuming the converter stays in DCM), v is the only state variable. From an approximate analysis, Tse found a map F: Å → Å of the form

$$v_{m+1} = \alpha v_m + \frac{\beta V_I (V_I - v_m)[\text{sat}\,(d_m)]^2}{v_m} \tag{19}$$

where α and β are constants involving f_s , L, C and R, and sat(·) is a saturation function that limits the duty factor so that $d_m \in [0, 1]$. The value of d_m was set by a proportional feedback scheme to $d_m = D - A(v_m - V)$, where D and V are the steady state (dc) components of d and v respectively. Using the gain A as the bifurcation parameter, a period doubling route to chaos was predicted, and confirmed by simulation using the exact equations. Experimental results were also reported.

The case where operation fluctuates chaotically between CCM and DCM would be interesting to study!

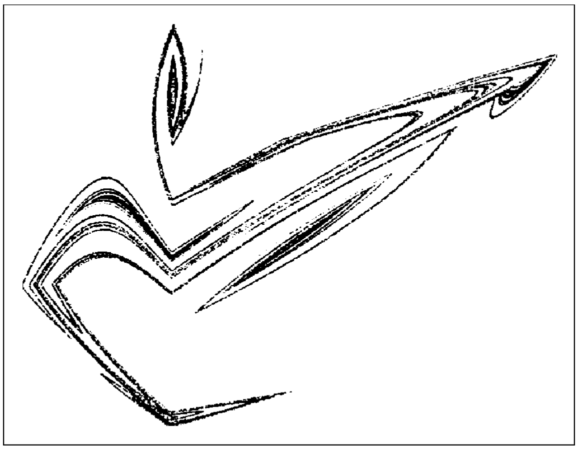

Fig. 7: Simulated Poincaré section of attractor for the buck converter in CCM with V_I = 35V.

10 Discussion

The state space averaging process has some evident flaws. First, all information about operation within a cycle is lost. Furthermore, the switching frequency f_s does not appear in the CCM model, though it must certainly have some effect. A subtler point is that d is purportedly a continuous-time variable; yet the duty factor is defined in terms of discrete time. Each switching cycle has an associated duty factor: it is meaningless to talk about changes in the duty factor within a cycle. The paradox becomes important with fast perturbations; it can be shown that the averaging process is exact when the perturbation frequency is zero, but is further in error the higher the perturbation frequency. In fact the natural sampling PWM imposes a Nyquist limit of $f_s/2$, beyond which the model is meaningless. Another minor problem is that the true duty factor is constrained to [0, 1], but the averaged variable d is not bounded (at least, not explicitly).

In both the CCM example and the DCM case studied by Tse, the conventional analysis using averaging is *qualitatively wrong*: it predicts stability for all input voltages, whereas in reality subharmonics and chaos are possible. Such a conclusion has worrying consequences where the reliability and safety of a system containing a buck converter is concerned.

Considerable effort has been expended to validate and improve upon the basic averaging process. Sanders et al. [52] developed a generalised averaging method with greater applicability; Krein et al. [53] considered Bogoliubov averaging; Tymerski applied the theory of time-varying transfer functions [54] and Volterra series [55]; variable structure systems theory (sliding mode control) was explored by Sira-Ramírez [56] and Bass [57]. These investigations build on sound theoretical bases, and usually “discover” state space averaging as the zero-order approximation, with higher terms giving more accurate results. Nonetheless, the simplistic averaging technique remains the most popular with practising power electronics engineers: it is easy to understand (if one does not probe too deeply), and straightforward to apply.

11 Simulation Issues

As with other nonlinear systems, computer simulation has a major role in investigations of power converters. However, the characteristics of switching circuits give rise to some distinctive problems [58].

To follow a trajectory numerically, the system of ordinary differential equations (ODEs) is solved by performing an approximate integration. For the general system $d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}, t)$, $\mathbf{x}(t = 0) = \mathbf{x}_0$, the trajectory is found by repeated application of

$$\mathbf{x}(t+h) = \mathbf{x}(t) + \int_t^{t+h} \mathbf{f}(\mathbf{x}, t)\, dt \tag{20}$$

where h is the time step, for some domain $t \in [0, t_{end}]$. Equation (20) can also be formulated as a Taylor series:

$$\mathbf{x}(t+h) = \mathbf{x}(t) + h\, \mathbf{f}[\mathbf{x}(t), t] + \frac{h^2}{2!}\mathbf{f}'[\mathbf{x}(t), t] + \frac{h^3}{3!}\mathbf{f}''[\mathbf{x}(t), t] + \cdots \tag{21}$$

Thus numerical integration of the ODE is equivalent to summing an infinite series. Two assumptions are usually made: 1) the solution $\mathbf{x}(t)$ is smooth (of class C^∞ over the domain $[0, t_{end}]$) so all the terms of the series exist; and 2) by choosing h sufficiently small, the series may be made to converge rapidly, so a few terms are sufficient for accuracy. Both assumptions are routinely violated by power electronic circuits.

Numerical errors are particularly important in chaotic operation, because any error, no matter how tiny, grows exponentially until it eventually dominates the solution. Nevertheless, it is still worth striving for accuracy: the smaller the error, the more closely the computed waveforms, bifurcation diagrams and attractors will resemble the true ones.

11.1 Discontinuous Right Hand Sides

As exemplified by (1), it is clear that the ODEs of ideal power converters have discontinuous right hand sides, i.e. $\mathbf{x}(t)$ is of class C^0: the derivative exists, but contains jump discontinuities at the switching instants. Therefore the first assumption of numerical integration is contravened: the derivatives in (21) do not exist for all $t \in [0, t_{end}]$. Because **f** is undefined at the switching instants, integrating across such a discontinuity (e.g. by using a fixed step size) is likely to incur a large error, even with a small value of *h*.

To circumvent this difficulty, power electronics simulators can take one of two actions. Switched circuit simulators determine the switching instant t_s accurately, then integrate up to t_s^-, apply the new value of **f** at t_s^+, set $\mathbf{x}(t_s^+) = \mathbf{x}(t_s^-)$ and integrate onwards. Unfortunately, there are still problems when one switching event leads to another: for example, in the buck converter, when S opens, D immediately starts conducting. Yet S and D must never conduct simultaneously, or infinite current would flow; nor must they block simultaneously or infinite voltage would be generated. Dealing with such situations automatically, without incorporating *a priori* knowledge of circuit operation, is fraught with difficulty [59].

The alternative is to replace the ideal switches with non-ideal ones. For example PSpice [60], a commercial development of the public domain circuit simulator SPICE [61], provides a switch model that has a non-zero on-resistance and a finite off-resistance, and which must transfer between on and off in non-zero time. The justification is that real switching devices behave in a similar way. The drawbacks to this approach are twofold: first, small time-constants are introduced, necessitating a stiff ODE solver (SPICE uses the trapezoidal method as standard), which, though stable, can introduce high frequency artefacts into the solutions; second, $\partial\mathbf{f}/\partial t$ is very large during switching transitions, causing very small values of *h*, and possible non-convergence of the inner iterations of the implicit ODE solver. (SPICE uses Newton–Raphson.)

11.2 Discontinuous Left Hand Sides

Unfortunately, matters are sometimes even worse. In certain circuits with ideal switches, $\mathbf{x}(t)$ is itself discontinuous! This can happen, for example, at the closing of a switch across a capacitance — perhaps the inherent capacitance of a switching device. If the capacitance *C* has an initial voltage $v \neq 0$, then an infinite current flows at the switching instant, dissipating energy of $\frac{1}{2}Cv^2$. To reduce such losses, a major class of power converters is designed so the switches close only when there is no voltage across them (*zero voltage switching* converters). Although this desirable condition may be obtained in the steady state, it may not extend to start-up and transient conditions.

Discontinuous left hand sides can be handled by switched circuit simulators if provision is made within the ODE solver to reset the state variables to their appropriate values: $\mathbf{x}(t_s^+) \neq \mathbf{x}(t_s^-)$. In SPICE-like simulators, a small time step must again be used to ensure accuracy during the transition. The price is that simulations take a long time if a slow transient is to be observed; run times of several hours are typical.

Computer simulations are a powerful tool for investigating nonlinear systems, but unfortunately the switched nature of power electronics causes some inherent numerical problems that cannot easily be sidestepped. Perhaps further development of the mathematics of discontinuous systems could help in this respect.

12 Some Possible Future Applications

There would seem to be two reasons for studying nonlinear dynamics in the context of power electronics:

- To understand better the nonlinear phenomena that occur in power converters, and thereby avoid undesirable effects.
- To allow converters to be engineered that deliberately make use of effects such as chaos.

Though the first objective has been achieved to some extent, there are as yet few practical power electronics applications in which subharmonics or chaos bring a distinct advantage. Nevertheless, with increasing awareness among power electronics practitioners of nonlinear dynamics, perhaps engineering uses will soon be found for nonlinear effects. It may be helpful to list the characteristics of chaos, and indicate some possible application areas.

Bounded erratic oscillation: The apparently random but bounded character of chaos suggests that it might be used in place of a pseudo-random generator. A possible application is on-line parameter identification. For example, vector control of induction motors requires a knowledge of the rotor time constant, but this varies because the resistance of the copper rotor winding changes with temperature. Pseudo-random sequences have been applied to identify the time constant while the motor is running; perhaps chaos could be used instead.

Broadband spectrum: Power converters produce interference concentrated at a harmonically related frequencies, and this may be undesirable. In drives that operate with switching frequencies in the audible range, acoustic noise may be produced and mechanical resonances excited. Pseudo-random generators have been employed to modulate the switching, spreading the acoustic energy over a wider spectrum and making it sound more acceptable (a hiss rather than a whine). Chaos may have a role to play here. Similarly, switching converters generate conducted and radiated electromagnetic interference at radio frequencies. Though matters can be improved with good design practice, filtering and shielding, it is difficult to meet international standards. Again techniques such as pseudo-random sequences and frequency modulation have been applied to spread the interference spectrum, and chaos is another possible contender. Peaks might be reduced by 10–15dB, less expensively than by other methods.

Sensitivity to perturbations, and control: The inherent sensitivity of chaotic systems to small perturbations may be exploited for synchronisation, targeting specific goals, and stabilising limit sets such as unstable equilibria or periodic orbits [62]–[68]. However, applications in power electronics are less obvious, because it is already possible to force large changes in behaviour by means of the active switching devices. Recent work demonstrates that chaotic power converters may also be stabilised by appropriate feedback [26], [27]. This begs the question: is there any point in making a power converter chaotic, in order to stabilise it? The answer is at present unknown, but such an approach may improve dynamic response. Fighter aircraft are designed to be open loop unstable but are then stabilised by feedback, making them more agile than conventional designs. Similarly, stabilised chaotic power converters may react more quickly, for instance in moving rapidly from one commanded output voltage to another. At present this suggestion is no more than speculation.

Validating nonlinear models: It has been noted that the bifurcation sequence of a nonlinear system is peculiar to that system, whereas two quite dissimilar systems may display superficially similar attractors [69]. Thus it is a requirement that a model of a nonlinear system should display the same bifurcation pattern as the original system. Such “fingerprinting” could prove a very powerful method of validating models of power converters and other nonlinear systems. It is noteworthy that excellent bifurcation fingerprint agreement has been obtained for the buck converter [20] and for the Jiles–Atherton model of a magnetic core [37], [38], suggesting that the modelling approaches adopted have good validity over a wide domain.

13 Conclusion

High efficiency solid state power conversion has become possible through the continuing development of high power semiconductor devices. The operation of these devices as switches, which is necessary for high efficiency, means that power electronic circuits are essentially nonlinear, time varying dynamical systems. Though this makes them difficult to study, the effort is well worthwhile because they have many practical applications and are becoming increasingly important in the delivery and utilisation of electrical energy. The conventional modelling approach effectively ignores nonlinear phenomena, and can sometimes mislead the designer into thinking a circuit will perform acceptably when in practice it will not. Thus the traditional approach does not always produce reliable models.

Discrete nonlinear modelling offers another way of looking at the circuits, one that is more accurate and able to reproduce nonlinear effects such as subharmonics and chaos. Unfortunately it demands a mental shift on the part of power electronics engineers, away from linear systems thinking and towards the unfamiliar realm of nonlinear dynamics [5], [70]–[72]. These techniques have not yet been widely adopted by power electronics practitioners, and there is much work still to be done.

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