Harmonic Analysis of the PWM Series/Parallel Bootstrap Variable Inductance

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Abstract — A recently introduced FACTS controller, the Bootstrap Variable Inductance, can emulate positive/negative variable inductance, for controlling either series or shunt parameters of transmission lines. A new analysis method for PWM of the BVI's switching amplifier is presented. A MATLAB program has been written implementing the formulas. The analytical results are confirmed for an example by FFT analysis, and the spectra are in good agreement. The analysis is also applicable to other PWM modulation schemes.

I. INTRODUCTION

The principal aim of *flexible ac transmission systems* (FACTS) is to control ac transmission parameters in order to govern active and reactive power while maintaining an adequate stability margin [1]. This can be done by adding reactance in series with the line, or in shunt at selected busbars. A new method has been proposed in [2] and [3], the Bootstrap Variable Inductance (BVI), for controlling either series or shunt parameters of transmission lines. Its series and parallel versions are called the SBVI and PBVI respectively. The BVI produces variable positive/negative inductance.

A. Inductance, Capacitance and Reductance

Conventionally, capacitors are used to compensate inductive reactance. In the new proposal, capacitance is replaced by a negative inductance, which for convenience is termed *reductance* [2]. If, for a circuit branch, $v = -\Gamma di/dt$, then Γ is the reductance (a positive value) of the branch, equivalent to a negative inductance of $L=-\Gamma$. Fig. 1 shows two cases comparing capacitance and reductance. For a sinusoidal input voltage (a single frequency ω_0), the capacitive and reductive currents are identical (Fig. 1(a)). For the distorted input voltage waveform of Fig. 1(b), the reductive current is much smoother than the capacitive current. Let $f(t)$ be a periodic waveform with period T and quarter-wave symmetry: $f(t) = f(t+T/4) = -f(t+T/2) = -f(t+3T/4)$. The Fourier series of the distorted input voltage can be expressed as $V_t = \sum_{n=1}^{\infty} A_n \sin \omega_n t$, and the capacitive current is

$$I_C(t) = C\frac{dV_C}{dt} = \sum_{n=1}^{\infty} CA_n\omega_n \cos\omega_n t \qquad (1)$$

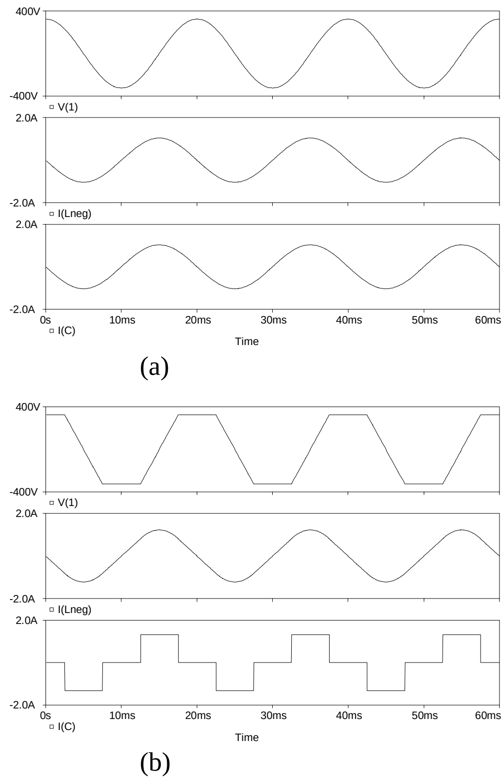


Fig. 1: Comparison of reductance and capacitance. (a) Sinusoidal voltage waveform, (b) a periodic non-sinusoidal voltage waveform. Top: impressed voltage; middle: current in reductance; bottom: current in capacitance. Although reductance and capacitance are indistinguishable at a single frequency, when harmonics are present their behavior is quite different.

where $\omega_n = n\omega_0$, C is the capacitance, and A_n is the nth-harmonic coefficient of the applied voltage; whereas the reductive current is

$$I_\Gamma(t) = \frac{-1}{\Gamma}\int_0^t V_\Gamma(t)dt + I_\Gamma(0)$$
$$= I_\Gamma(0) + \sum_{n=1}^{\infty} -\frac{A_n}{\omega_n\Gamma}(1-\cos\omega_n t) \qquad (2)$$

$I_\Gamma(0)$ can be chosen so that the dc term vanishes. Comparing (1) and (2), the high-order harmonic currents are much greater in the capacitance than in the reductance. Therefore we believe reductance is more appropriate than capacitance if distortion is present. Additionally, capacitance will resonate with inductance whereas reductance will not. So, by using reductance in power systems the potential risk for sub-synchronous resonance is greatly reduced.

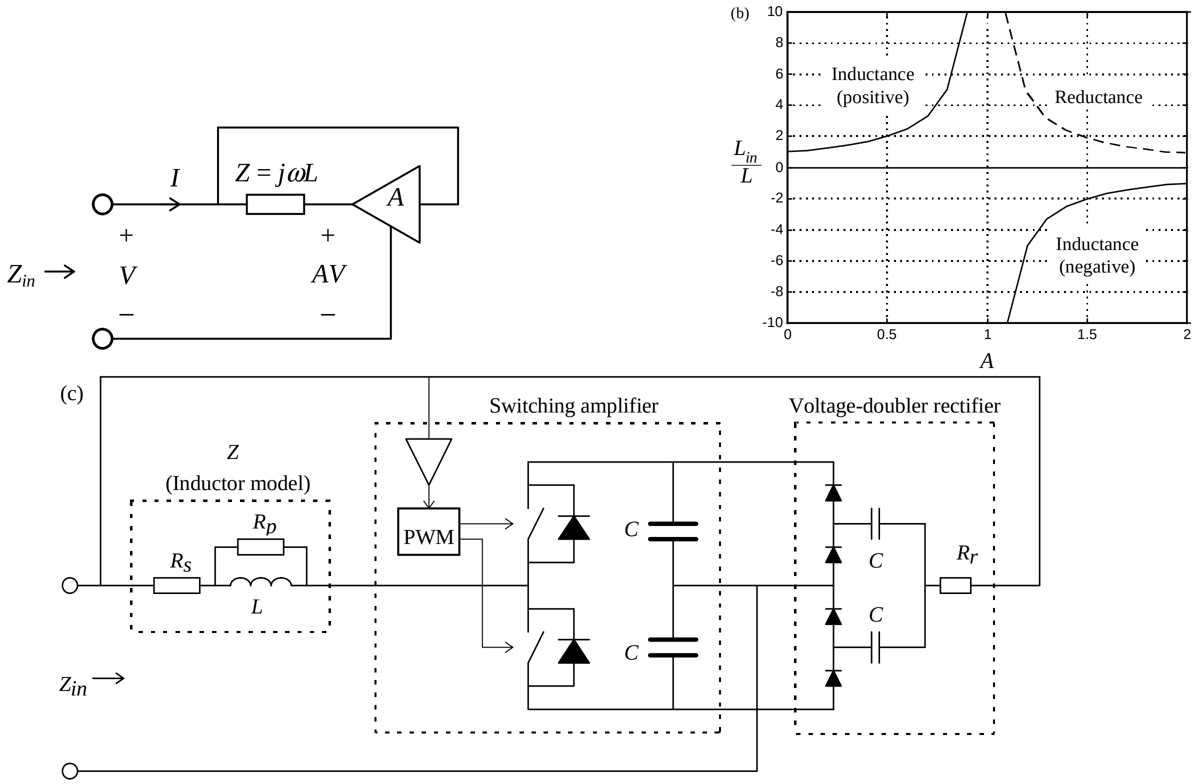


Fig. 2: (a) Principle of variable inductance/reductance, (b) range of emulated inductance L′ as a function of amplifier gain A, (c) BVI circuit schematic, showing a possible implementation with self- powering from the applied voltage.

B. *PWM Analysis*

In this paper we concentrate on the control of the BVI's switching amplifier. In [4], a switched-mode output stage with pulse-width modulation (PWM) has been proposed for FACTS applications. Ideally, the modulation scheme should be designed to minimize the harmonics introduced into the power system. To achieve this, the higher the switching frequency, the lower the harmonics but also the higher the switching losses. Therefore a compromise must be reached, and we need a way of evaluating the harmonic structure of the PWM waveform for a given carrier frequency. We have developed a new analysis method for PWM aimed at the BVI's switching amplifier. However, the proposed method can be applied to any PWM modulation scheme that satisfies the assumptions listed below.

In this paper, we review the BVI in Section II, and the necessary formulas for the proposed method are derived in Section III. A program has been developed with MATLAB, and is included in Appendix I. The results from the proposed method are compared with those from the Fast Fourier Transform (FFT) in Section IV, and an assessment of the BVI's output is provided. The effect of the amplifier's gain on the fundamental and harmonics is examined. Additionally, the total harmonic distortion of the BVI's amplifier output is discussed, together with advantages and disadvantages of applying PWM to the BVI.

II. REALIZATION OF THE BVI

Negative inductance was first proposed as a power-system series compensator in 1992, in the form of a "variable active-passive reactance" (VAPAR) [5], and this circuit has subsequently been developed [6]–[10]. Recently we introduced a new method, the BVI, in [2] and [3] for both parallel and series applications in power systems. Fig. 2(a) shows the principle, which is based on feedforward. An impedance Z in series with a variable gain amplifier $A(j\omega)$ provide a variable input impedance Z':

$$Z' = \frac{V}{I} = \frac{Z}{1 - A(j\omega)} \qquad (3)$$

where V and I are input voltage and current phasors.

Now let $A(j\omega) = A$ be a positive constant. When $A < 1$, Z' has the same sign as Z but greater magnitude. When $A = 1$, $Z' = \infty$ so $I = 0$. This principle is known in analog electronics as bootstrapping. When $A > 1$, Z' has the opposite sign to Z (negative impedance conversion). Thus if we make Z inductive ($Z = j\omega L$), the circuit will emulate a reductance ($Z' = -j\omega\Gamma$). Therefore, by varying A, a wide range of inductance and reductance can be emulated ($L' = L/(1 - A)$, as shown in Fig. 2(b).

Fig. 2(c) shows a single-phase BVI circuit. The capacitors are fed by a voltage-doubler rectifier which is supplied by the applied voltage. The applied voltage also forms the reference for the PWM, whose output drives two switches. In the next section we focus on the PWM control of these switches.

III. PWM SCHEME AND ANALYSIS

The PWM block, shown in Fig. 2(c), uses the applied voltage as its reference waveform. (Alternatively, an external reference could be used.) The two ideal switches are driven so that when one of them is closed the other is opened. However, in practice the anti-parallel diodes would conduct during a short gap when both switches are open, to avoid overlap. Also, in every half-cycle of the applied voltage, based on the switching frequency, one switch and anti-parallel diode of the other switch carry the input current. In other words, each switch carries current for a half cycle of synchronous frequency. Therefore, the PWM provides a bi-level waveform at the output of the amplifier, denoted as a sequence of pulses at levels of 1 and –1 times the capacitor voltage levels.

Although various modulation strategies are possible, this discussion is limited to naturally-sampled PWM with two special groups of carrier waveform:

(i) those that intersect with the reference waveform at exactly one point in each carrier period (e.g. a ramp carrier and a sinusoidal reference);

(ii) those that intersect at exactly two points (e.g. a triangle carrier and a sinusoidal reference).

A. Problem Definition

We make the following assumptions:

1. A periodic carrier waveform $V_C(t)$ with period T_C (this is our choice)
2. A periodic reference waveform $V_R(t)$ with period T_R (this is the power system constraint)
3. $T_R = N\,T_C$

The following function is defined according to the comparison performed within the PWM block:

$$f(t) = \begin{cases} 1 & if \quad V_R(t) \geq V_C(t) \\ -1 & if \quad V_R(t) < V_C(t) \end{cases} \qquad (4)$$
$$= \mathrm{sgn}(V_R(t) - V_C(t))$$

The objective is to find a Fourier series for the given function *f(t)* in the form of

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \qquad (5)$$

where $\omega_0 = 2\pi/T_R$, $F_n = (a_n - j\,b_n)/2$, and a_n, b_n are sinusoidal and cosinusoidal Fourier series coefficients of $f(t)$ respectively.

B. Lower Bound on N

We first find a conservative bound on the value of *N*. We show that, to ensure only one (case i) or two (case ii) intersections within each T_C, in the worst case the carrier frequency must be at least four times the reference frequency.

Case (i): Let the carrier $V_C(t)$ be a rising ramp between $[t_0, t_0+T_C]$ increasing from -1 to 1. Thus:

$$V_C(t) = \frac{2}{T_C}(t - t_0) - 1 \qquad (6)$$

Let the reference $V_R(t) = k\sin\omega_S t$, where *k* has a range of [0,1]. As we are interested in intersections of the carrier and reference, by defining $V_R(t) - V_C(t)$,

$$V_{RC}(t) = k\sin\omega_S t - \frac{2}{T_C}(t - t_0) + 1 \qquad (7)$$

we can find the intersections by zero-crossing $V_{RC}(t)$. It is clear that if $V_{RC}(t)$ has no maximum or minimum within the interval, then there can only be one intersection. At the same time, if the number of maximums are equal to the minimums and all of them are in one side of the *t* axis, still there is only one intersection. However, we restrict the discussion to an ascending and/or descending $V_{RC}(t)$, which causes higher extreme on *N*.

By equating the derivative of $V_{RC}(t)$ to zero, it gives $\cos\omega_S t = \frac{T_R}{k\pi T_C}$ which is less than one. This results $N = \frac{T_R}{T_C} \leq k\pi$, if there is to be the possibility of more than one intersection. To avoid more than one intersection, therefore, $N \geq k\pi$. As *k* is less than 1 (full modulation: *k* = 1), If *N* is integer, then in the worst case:

$$N = 4, 5, 6, \ldots \qquad (8)$$

This conservative lower bound is easily met, since the carrier frequency for power systems is greater than 200 Hz.

Case (ii): A triangular carrier waveform can be constructed from a rising ramp of duration mT_C and a falling ramp of duration $(1-m)T_C$,where $m<1$. (E.g. *m* = ½ for a symmetrical triangle wave.) If each ramp

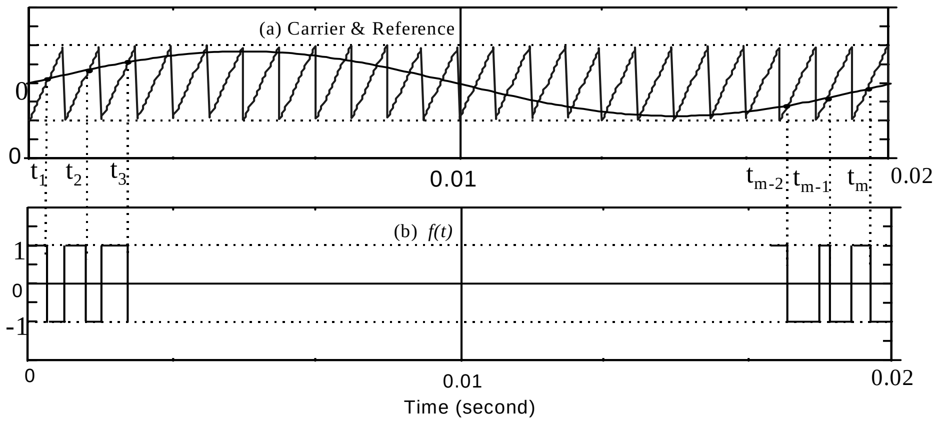


Fig. 3: Definition of the PWM through an example: (a) ramp carrier and sinusoidal reference, (b) $f(t)$ defined in equation (4). In this example $\boldsymbol{t_1, t_2, t_3, \ldots, t_{m-2}, t_{m-1}}$, and t_m are intersection points of ramp carrier and sinusoidal reference.

intersects with $V_R(t)$ at exactly one point, the above analysis remains valid, provided T_C is replaced by mT_C for the rising ramp and $(1-m)T_C$ for the falling ramp. For the rising ramp we find $N = \dfrac{T_R}{T_C} \le km\pi \le \pi$, if multiple intersections are to occur, i.e. choosing N based on (8) will avoid them. For the falling ramp, $N = \dfrac{T_R}{T_C} \le k(1-m)\pi \le \pi$, and we must obey (8) again. If k and m are known, the bound can be improved; e.g. if k = 1, and m = 1/2, $N \ge 2$.

C. Formula Derivation

In our solution, first of all, the intersections of the carrier and reference waveform are obtained. Let us assume these points as t_1, t_2, t_3, …, t_m. Fig. 3(a)–(b) shows this intersection and the resultant function $f(t)$ defined in equation (4). By definition, the nth Fourier coefficients are:

$$a_n - jb_n = \frac{2}{T_R}\int_0^{T_R} f(t)e^{-jn\omega_o t}\,dt \tag{9}$$

Now, we consider the first type of carrier: those which intersect with the reference at exactly one point during T_C. The nth Fourier coefficients are:

$$a_n - jb_n = \frac{2}{T_R}\sum_{k=1}^{m}\Big(\int_{(k-1)T_C}^{t_k} e^{-jn\omega_o t}dt - \int_{t_k}^{kT_C} e^{-jn\omega_o t}dt\Big) \tag{10}$$

Calculating the above integrals and simplifying the results:

$$a_n - jb_n = \frac{2}{-jn\pi}\sum_{k=1}^{m} e^{-jn\omega_o t_k} - e^{-jn\omega_o (k-1)T_C} \tag{11}$$

The dc component of $f(t)$ is:

$$a_o = \frac{2(t_1 + t_2 + t_3 + \cdots + t_m) - m^2 T_C}{T_R} \tag{12}$$

For the second case, those carriers which intersect with the reference in exactly two points during T_C, the nth Fourier coefficients are:

$$a_n - jb_n =$$
$$\frac{2}{T_R}\sum_{k=1}^{m}\Big(\int_{t_{2k-2}}^{t_{2k-1}} e^{-jn\omega_o t}dt - \int_{t_{2k-1}}^{t_{2k}} e^{-jn\omega_o t}dt + \int_{t_{2k}}^{T_R} e^{-jn\omega_o t}\,dt\Big) \tag{13}$$

Again, calculating the integrals and simplifying the results:

$$a_n - jb_n = \frac{2}{-jn\pi}\sum_{k=1}^{m} e^{-jn\omega_o t_{2k-1}} - e^{-jn\omega_o t_{2k}} \tag{14}$$

The dc component of $f(t)$ is:

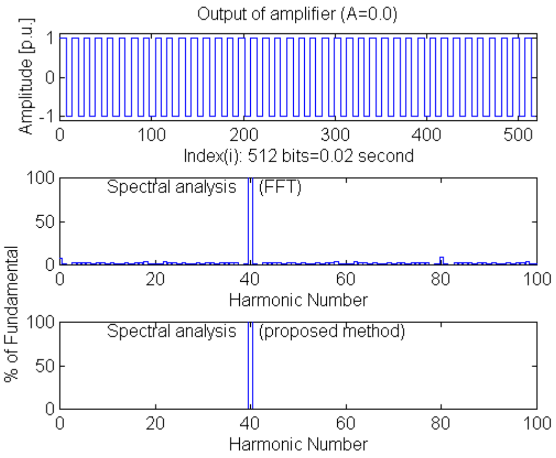
$$a_o = \frac{2(t_1 - t_2 + t_3 - t_4 + \cdots + t_{2m-1} - t_{2m}) + T_R}{T_R} \tag{15}$$

Equations (11) and (14) are the main results of this paper. They have been evaluated with a program and are assessed in the next section.

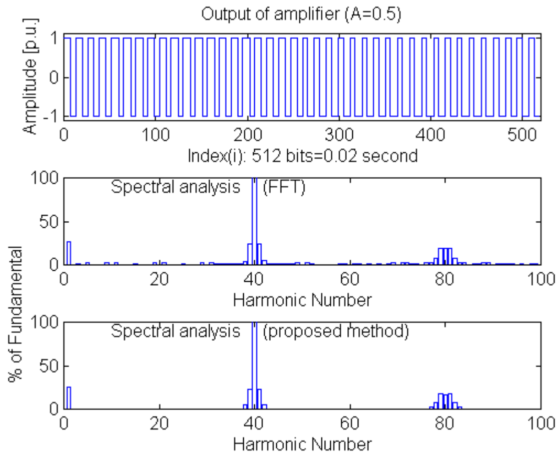
IV. ASSESSMENT OF THE RESULTS

Considering (3), the BVI's inductance/reductance can be varied by changing the amplifier gain A. This is made possible by varying the amplitude of the reference waveform of the PWM. Based on (11) and (14), the spectral analysis of $f(t)$ has been programmed. To validate the analysis, the results have been compared with those from the FFT (which is slower and less accurate).

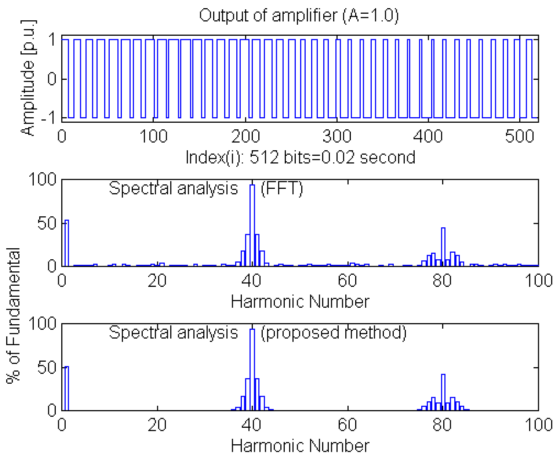
Fig. 4(a)–(e) shows the output of PWM-controlled amplifier (per unit by the input voltage), the spectral analysis performed with the software developed, and the spectral analysis made by FFT for A = 0.0, 0.5, 1.0,.1.4, 2.0. The carrier is a ramp of 2kHz and the reference is a 50Hz sinewave. As can be seen, the results are almost the same, confirming the validity of the proposed method. The small amount of noise in the FFT results is due to windowing.



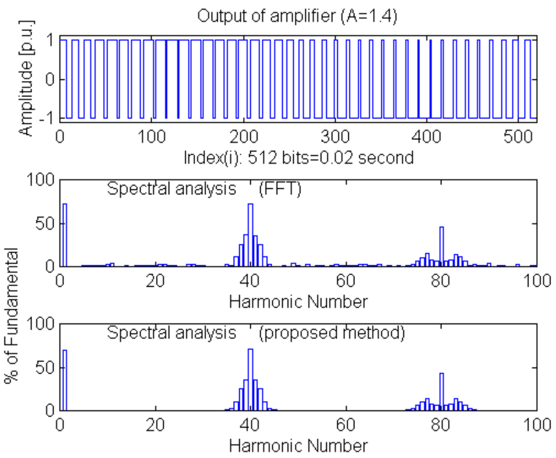
(a)



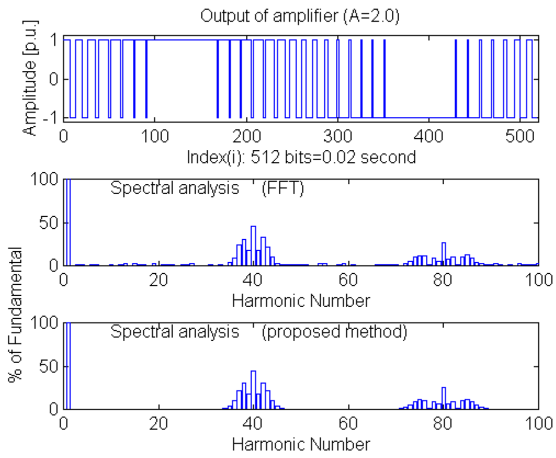
(b)



(c)



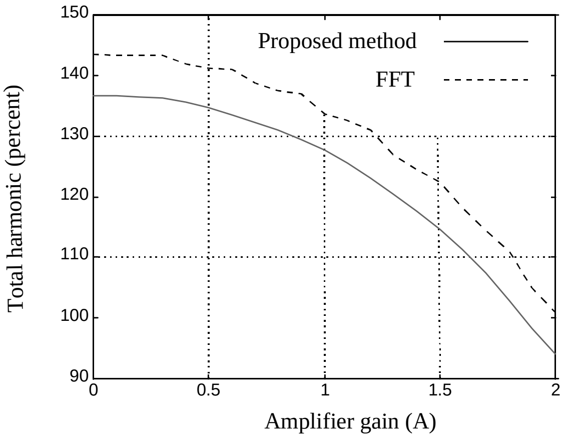
(d)

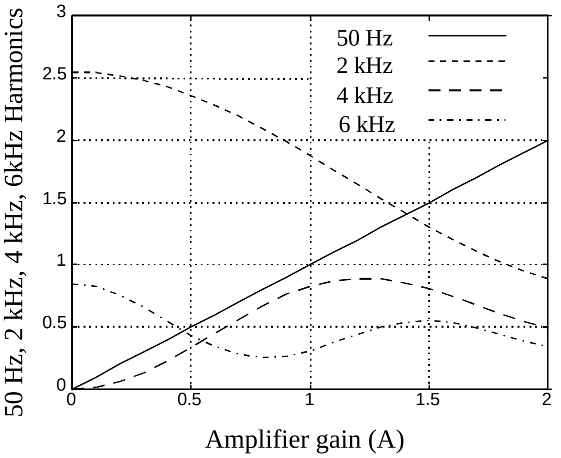


(e)

Fig. 4: (a)–(e): results from the proposed PWM analysis method and the FFT. Upper waveform is the amplifier output in per unit, the middle and lower waveform are spectra taken from FFT and the proposed method respectively.

As the window time increases, the noise should reduce. (The FFT implements a Fourier transform whereas the proposed analysis uses a Fourier series.)





(b)

Fig. 5 (a) The total harmonic percentage of the amplifier output (normalized by the BVI's input voltage) versus its gain and comparison between two methods; (b) calculated harmonics (normalized by the BVI's input voltage) against the amplifier gain: 50Hz fundamental, 2kHz switching frequency, 4kHz and 6kHz switching-frequency harmonics.

Fig. 5(a) shows the variation of total harmonic percentage (normalized by the BVI's input voltage) versus the amplifier gain. Two curves correspond to the FFT and the analysis. The total harmonics of the amplifier output are greater than 85% according to the developed software. Fig. 5(b) shows the variation of fundamental and three multiples of the switching frequency harmonic (normalized by the input voltage) versus the amplifier gain A. These normalized harmonics are considerable, especially for the switching frequency (2kHz). For example, the 2kHz harmonic for A = 0.0 is more than 2.5 times the input voltage magnitude, affecting the input current of the BVI. This implies that filtering the switching frequency and its harmonics is necessary for the BVI.

V. CONCLUSION

PWM for the SBVI/PBVI, newly suggested FACTS controllers, has been analysed. The results from the proposed method have been compared with those from a conventional FFT and their validity confirmed. Although the main objective of the analysis is the switching control of the BVI, it can also be applied to other power converters in which the PWM carrier intersects the reference waveform at exactly one point or two points in each carrier period.

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