CHAOS IN A SPACECRAFT POWER SYSTEM

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Abstract: A simple model of a spacecraft dc power system is investigated using the techniques of nonlinear dynamics. Limit cycles, coexisting attractors and chaos are observed. Chaos is confirmed by a positive Lyapunov exponent. The results have implications for reliable spacecraft engineering.

Introduction: It is established that nonlinear phenomena, including undesirable bifurcations and chaos, can occur in power converters [1] and ac power systems [2], but so far these effects have not been demonstrated in a dc power system of practical value. Spacecraft such as earth-orbiting satellites universally employ dc power systems, which are mission-critical: malfunction or failure can render the entire spacecraft unusable. In this Letter we investigate a simple model, Fig. 1, which contains the elements of a basic space power system, and demonstrate the possibility of chaos.

The model system of Fig. 1 comprises a solar array, a low-pass filter, an undervoltage lockout, a dc-dc converter, and a payload. Real spacecraft are more complex, including a battery and additional converters, but we study this minimal system for simplicity. Refer to Fig. 1 for symbols.

Solar array: The solar array characteristic

$$i_1 = \alpha\, I_{sc}\left(1 - \frac{\exp(v_1/V_{th}) - 1}{\exp(V_{oc}/V_{th}) - 1}\right)$$

is derived from the well known exponential model of a photovoltaic cell. Here I_{sc} is the array's short-circuit current, V_{oc} is its open-circuit voltage, and the array thermal voltage $V_{th} = mkT/q$, where there are m cells in series, k is Boltzmann's constant, T is the absolute temperature and q is the electronic charge. We assume $i_1 \in [0, I_{sc}]$ and $v_1 \in [0, V_{oc}]$ (normal operation). Parameter $\alpha \in [0, 1]$ denotes the relative solar illumination, which varies with the orbital position and attitude of the spacecraft. It is used here as a bifurcation parameter.

Low-pass filter: The filter comprises a capacitance C_1, which includes the solar array's self-capacitance; an inductance L, which includes wiring inductance; a resistance R, which represents parasitic losses; and a capacitance C_2, which includes the input reservoir capacitance of a practical dc-dc converter. The state equations are

$$\begin{aligned} dv_1/dt &= (i_1 - i_L)/C_1 \\ di_L/dt &= (v_1 - i_L R - v_2)/L \\ dv_2/dt &= [i_L - i_2\, s(t)]/C_2 \end{aligned}$$

where $s(t) \in \{0, 1\}$ is the state of switch S at time t.

Undervoltage lockout: Switch S disconnects the dc-dc converter if there is insufficient input voltage to produce a regulated output. (Such circuits are also commonly used to prevent bus voltage collapse and battery over-discharge.) The switch opens if v_2 falls below a lower threshold V_{lo} and closes if v_2 rises above an upper threshold V_{hi}. This hysteresis, which is intended to prevent switch chattering, is described by a switching function $s(t)$:

$$s(t^+) = \begin{cases} 0 & \text{if } v_2 \le V_{lo} \quad \text{(switch open)} \\ 1 & \text{if } v_2 \ge V_{hi} \quad \text{(switch closed)} \\ s(t^-) & \text{if } V_{lo} < v_2 < V_{hi} \quad \text{(switch unchanged)} \end{cases}$$

Dc-dc converter and payload: The converter is modelled as ideal: it is lossless, ripple-free and has an internal controller which is fast enough that the converter dynamics can be neglected. The converter supplies a payload which draws constant power P. The converter's input characteristic is $i_2 = P/v_2$.

Simulation results: Combining these models, we applied a Runge–Kutta fifth-order six-stage integration routine to find the state-space trajectories numerically. A fixed time step was used except near the switching instants, which were located precisely via repeated bisection. (This is necessary to minimise truncation error.)

This system is nonlinear and of third-order: these are necessary but not sufficient conditions for chaos. It is generally accepted that a positive Lyapunov exponent is the best proof of chaos. Computation of Lyapunov exponents in non-smooth systems requires special handling of discontinuities, and we employed a modified variational-equation algorithm [3] to ensure correct results.

The following parameter values were used: I_{sc} = 4A, V_{oc} = 46.2V, V_{th} = 1.1385V, C_1 = 50μF, L = 700μH, C_2 = 350μF, V_{lo} = 19V, V_{hi} = 21V, P = 50W, and an integration time step of 50μs. The bifurcation diagrams of Figs. 2(a) and (b) plot $\{v_1\} \mid v_2 = V_{hi}$ for $\alpha \in [0.3, 0.6]$, while Fig. 2(c) shows the largest Lyapunov exponent, λ_{max}. Figs. 2(a) and (b) were computed from different initial conditions: for $\alpha \in [0.3, 0.443]$ two limit cycles ($\lambda_{max} = 0$) coexist, but at $\alpha = 0.443$, that of Fig. 2(b) becomes unstable. For $\alpha \in [0.547, 0.597]$ chaos ($\lambda_{max} > 0$) is interspersed with periodic windows. There are stable equilibrium points ($\lambda_{max} < 0$) for $\alpha \approx 0.592, 0.595$. When $\alpha > 0.597$ the switch closes continuously, giving a stable equilibrium which is the intended operating mode of the power system. Fig. 3 shows a Poincaré section, $\{(v_1, i_L)\} \mid v_2 = ½(V_{lo} + V_{hi})$ for $\alpha = 0.576$, within the chaotic region. The fractal nature of the attractor is further evidence of chaos.

Conclusion: Chaos has been demonstrated in a model spacecraft power system by means of bifurcation diagrams, a positive Lyapunov exponent and a fractal attractor. Care was taken to ensure that the numerical methods themselves were not responsible for these phenomena. Although the results shown are based on a single set of parameters, chaos was also observed with other values.

Designers should recognise that limit cycles, coexisting attractors, bifurcations and chaos can occur even in a simple power system. If such nonlinear behaviour is left unpredicted or unrecognised, it could lead to malfunctions or even mission termination. In spacecraft engineering, which is characterised by complex nonlinear multi-dimensional power systems, there are important implications for reliability.

REFERENCES

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FIGURE CAPTIONS

Fig. 1: Simple model of a spacecraft power system.

Fig. 2: (a) Bifurcation diagram of $\{v_1\}$ against relative illumination level α, with initial conditions $v_1 = v_2 = 20.9$V, $i_L = 1$A;

(b) as (a), but with $v_1 = v_2 = 20.0$V;

(c) largest Lyapunov exponent λ_{max}.

Fig. 3: Poincaré section for $\alpha = 0.576$, within the chaotic region, revealing a fractal attractor.

FIGURES

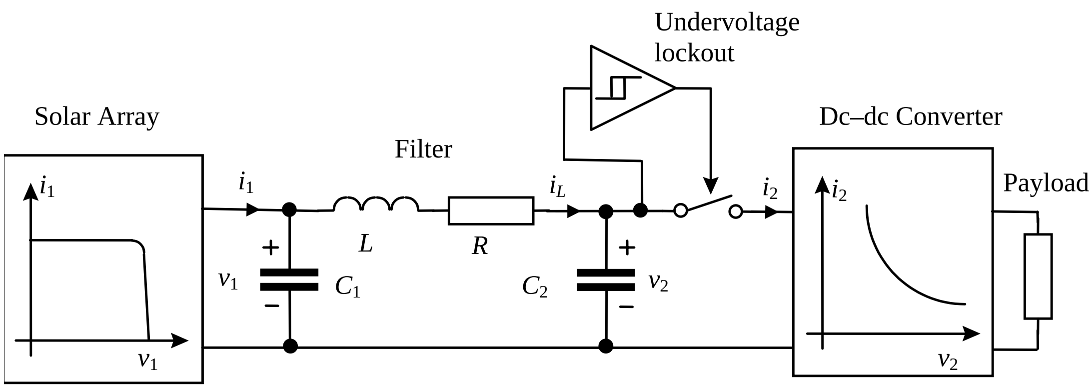

Fig. 1

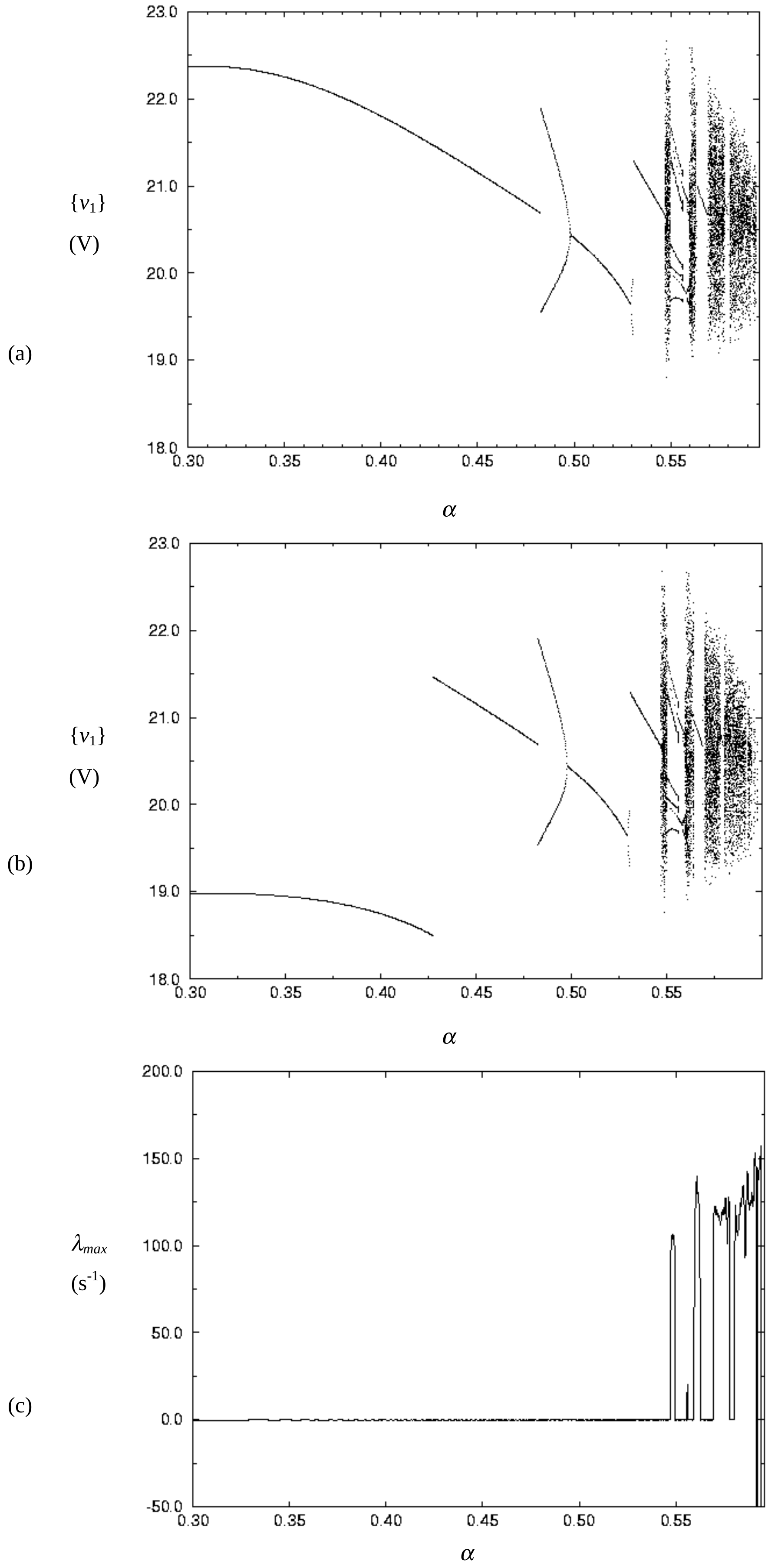

Fig. 2 *Note to Editor: please set this as a single tall figure if possible.*

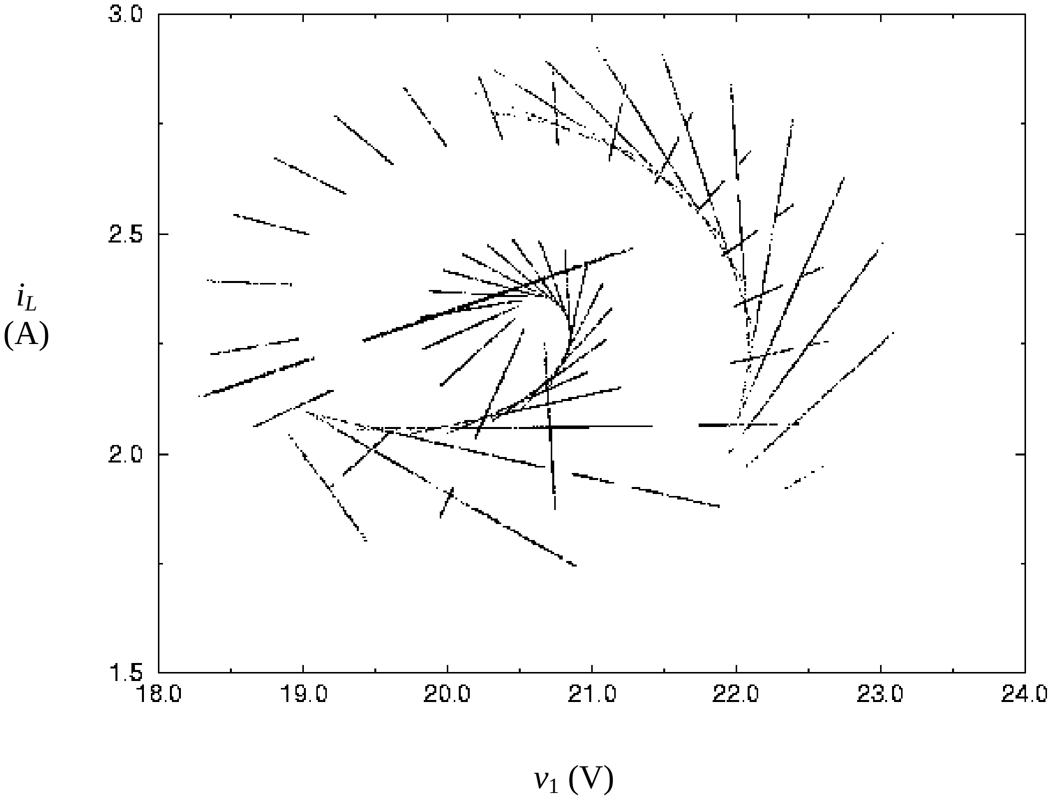

Fig. 3